Entry Costs and Employment Fluctuations in Frictional Labor Markets

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Abstract

Firms pay fixed costs upon entering labor markets and before recruiting workers. Modern theory of unemployment such as Pissarides (1985, 2000) posits that firms pay no such costs upon entry and vacant firms pay flow recruiting costs after entry. I find that introducing such fixed entry costs into otherwise standard matching models of unemployment will improve the models’ performance in two different dimensions. First, the model with entry costs delivers much larger impacts of productivity changes on the vacancy unemployment ratio than the standard matching model with flow vacancy costs only. Second, the extended model with flow recruiting costs and schooling choices predicts the weakly procyclical school attendance, which is at odds with the existing empirical findings. The model with entry costs and schooling decisions can be consistent with the countercyclical school attendance.

JEL classifications: E24; E32; J41; J63; J64

Key Words: Search and matching; Vacancies and unemployment; Recruiting costs; Entry costs; School attendance

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1I am grateful for comments from Robert Shimer and seminar audiences at various places. This research grows from chapters of my Ph.D. dissertation, submitted to the University of Chicago. In particular, the idea of introducing the fixed costs of firms’ entry into search models is examined in different contexts in Chapter 3 of my Ph.D. thesis. All errors are my own. This work is supported by MEXT, ACADEMIC FRONTIER (2006-2010). Financial support from the University of Chicago and the John Olin Foundation is gratefully acknowledged.

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1 Introduction

Firms pay fixed costs upon entering markets and before recruiting workers: the purchase cost of machines or capital, the cost of designing teams and organizations and so on. Modern theory of unemployment such as Pissarides (1985, 2000) posits that firms pay no such cost upon entry and a vacant firm pay flow recruiting costs after entry. I explore implications of introducing such fixed entry costs into otherwise standard matching models of unemployment. I find that such a model can deliver much larger impacts of productivity changes on the vacancy unemployment ratio than the standard matching model with flow vacancy costs.

There is a reason that motivates me to consider an alternative specification of firms’ free entry. The post-war U.S. data of unemployment and corporate profits, along with two key equations in matching models of labor markets (steady state accounting and free entry of vacancies), suggest that the variations in corporate profits can determine some, but not all of the, variations in the unemployment rate. Many recent authors have made efforts in incorporating mechanisms of generating wage rigidity in frictional labor markets.\(^3\) If incorporating wage rigidity into matching models is the key to understanding fluctuations of unemployment, the empirical fluctuations of profits should be able to explain fluctuations of unemployment, when the profits are fed into free entry equations. However, as far as I know, nobody has ever checked this. I start my exploration by putting the key equation in matching model — free entry equations— to data of unemployment and profits in the postwar U.S. I find that the behavior of corporate profits divided by GDP explains some, but not all, of the variations in unemployment rates.

\(^3\)Costain and Jansen (2006) incorporates worker-shirking problems into matching models, although their emphasis is on implications for cyclical nature of endogenous match separation rates. Kennan (2005), Moen and Rosen (2005) and others incorporate asymmetric information into matching models and explore its implications for wage rigidity.
The results of this paper is summarized by Figure 1 below. The flattest curve corresponds to the relation between productivity and the model-generated vacancy unemployment ratio for the standard matching model with flow vacancy costs. The vacancy unemployment ratio increases by about 1.7 percent as productivity increases by one percent. As is well known, the model-generated impact of productivity changes on the vacancy unemployment ratio is much smaller than data (Hall 2006; Shimer 2006). Five different curves in the Figure 1 correspond to matching models with fixed entry costs. Figure 1 says that the impact of productivity changes on the vacancy unemployment ratio depends on the rate at which vacancies break. More precisely, the less likely vacancies break, the larger the impact of productivity changes on the vacancy unemployment ratio generated by such models. In standard matching models of unemployment in which firms incur no entry costs and vacancies incur flow recruiting costs, the rate at which vacancies break does not matter: the value of vacancies is zero in equilibrium. I also add aggregate productivity shocks to such matching models and compute the size of fluctuations of the vacancy unemployment ratio generated by such models, into which productivity shocks of an empirical magnitude are fed. I examine and quantify different models: a model with flow vacancy costs and a model with fixed entry costs. In one model with fixed entry costs, both vacancies and filled firms break at the same rate. In the other model with fixed entry costs, vacancies do not break. Differences in the rate at which vacancies break have important implications for the size of fluctuations of the vacancy unemployment ratio generated by such class of models.
FIGURE 1: The vacancy unemployment ratio as a function of productivity

Notes: \( s_v \) is the rate at which vacancies break.

This finding raises the following two important questions: (i) Which specification of entrepreneurs’ costs before production is more plausible? ; and (ii) What is the sensible value of \( s_V \), the rate at which a vacant position disappears ?. I stress that answering these questions is important to make my argument of introducing entry costs more convincing. Since I am aware of no direct evidence of entrepreneurs’ entry costs and vacant positions’ destruction rate, my strategy here is to enrich the model slightly so that it delivers different kinds of predictions, which can be compared with data. To do so, I add choices of time-consuming schooling into otherwise standard matching models of unemployment. Time-consuming schooling is necessarily non-decreasing in productivity levels when firms incur only flow recruiting costs while being vacant. However, it can be either increasing or decreasing in productivity levels when firms incur fixed entry costs. Since the unemployment rate is decreasing in worker productivity levels, this finding suggests that the model with flow vacancy costs only predicts the weakly negative
relation between the unemployment rate and school attendance: weakly procyclical school attendance. However, a number of studies suggest that school attendance at the level of community college level is strongly countercyclical (Betts and McFarland 1995; Dellas and Sakellaris 2003; DeJong and Ingram 2001). The model with entry costs is consistent with the empirical findings, since it predicts either positive or negative relations between the unemployment rate and the school attendance\(^4\). The empirical studies cited above also provides disciplines in choosing the plausible value of \(s_V\), the rate at which each vacant position disappears.

**Literature**

A number of papers have assessed matching models of frictional labor markets. Hall (2003, 2005), Shimer (2005) and others argue that a matching models of Diamond, Mortensen and Pissarides, cannot deliver fluctuations of vacancies and unemployment of an empirically plausible magnitude when one feeds productivity shocks of the empirical size into the model. Hagedorn and Manovskii (2005) criticize calibration practices undertaken by these authors, by arguing that important parameters such as workers’ bargaining power and job-seekers leisure value, are not calibrated properly by matching models’ predictions to data. The mechanism explored in my paper is slightly related to the one in Hagedorn and Manovskii. They argue that the appropriate value of job-seekers’ flow utility should be much larger than the one used in Shimer and others’ calibrations. I show that introducing entry costs into otherwise standard matching models of unemployment is isomorphic to adding flow capital costs into such models. With flow capital costs considered, the flow profits will be reduced by the size of flow capital costs. The effect of adding flow capital costs on free entry equations is similar to the one of increasing

\(^4\)Manski and Wise (1983) find that enrollments in four-year college is not significantly related to the local unemployment rate. A number of studies, finding the statistically significant positive relation between enrollements and the unemployment rate, look at two-year community college levels. I argue in the main text later that this is also consistent with the extended matching model with entry costs and schooling choices.
the size of job-seekers’ flow utility.

The idea explored in this paper is related to the brief remark made in Mortensen and Nagypal (2006). They argue that introducing flow capital costs into otherwise standard matching models of unemployment will amplify the size of fluctuations of the vacancy unemployment ratio generated by such models. However, they are silent about how their flow capital costs are related to entry costs. Moreover, they are silent about the way in which differences in the rate at which vacancies break affect the size of the impact of productivity changes on the vacancy unemployment ratio generated by such models. Most importantly, my paper provides an alternative reason for why introducing entry costs into otherwise standard matching models of unemployment is plausible: The model with flow vacancy costs only is at odds with the empirically observed positive relations between the school attendance and the unemployment rate (Betts and McFarland; Dellas and Sakellaris; DeJong and Ingram); and the model with entry costs is consistent with the existing empirical works mentioned above.

The structure of the paper goes as follows. The next section undertakes some simple exercise using data of corporate profits and unemployment and free entry equations. Section 3 builds variants of matching models in which potential firms incur fixed costs upon entry and no recruiting costs while being vacant. Section 4 reviews a DMP model with flow recruiting costs. Section 5 examines the economy with aggregate shocks to productivity. Section 7 examines time-consuming schooling choices in a matching model of unemployment. Section 7 concludes.

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5I am grateful to Robert Shimer for bringing my attention to this paper.

6Sections 3 and 6 are main parts of the paper. The reader can skip the rest of the Sections without missing the main message of the paper.
2 Inspection of Free Entry Equations: Do Profits Determine Unemployment?

Free entry of vacant jobs is the key mechanism behind matching models of frictional labor markets. The matching model posits that employers post vacant firms so that they exhaust all profits from creating jobs. Free entry conditions say that:

\[ k = \mu \theta^{-\alpha} J \Leftrightarrow \theta^\alpha = \frac{\mu}{k} J \]

where \( k > 0 \) is a flow recruiting cost, \( \mu \theta^{-\alpha} \) is the rate at which a vacant job meets a job-seeker, and \( J \) is the value of a filled job, which is a present discounted value of firms’ profits. An inability of matching models to deliver large fluctuations of \( \theta \) can be traced to either small fluctuations of profits \( J \) or small variations in \( \mu/k \). I examine what would be behavior of \( \theta \) if profits were actual corporate profits in data, using the free entry equation and steady state accounting equation:

\[ u = \frac{s}{s + \mu \theta^{1-\alpha}} \]

This simple exercise is helpful to see whether variation in profits determine the variation in unemployment. If the answer is yes, one can say that introducing wage rigidity may be the key to improve the models’ ability to deliver large fluctuations of unemployment. If the answer is negative, one may need to consider an alternative setup of free entry of vacancies for the model to able to deliver unemployment fluctuations in data.

Let a free entry equation be specified as \( \mu \theta^{-\alpha} J = k \), where \( J \) is the present discounted value of profits, constructed from data of corporate profits in the U.S. 1948-2000. Let quarterly interest rate \( r \) be 0.012, and quarterly separation rate \( s \) be 0.1. I pick vacancy cost parameter \( k \) so that predicted \( \theta \) in 2000 is equal to 1 (normalization). I pick matching function parameter \( \mu \)
so that the theoretical unemployment rate \( u \) is equal to data in 2000. Thus, I choose \( \mu = 1.9 \) and \( k = 1.52 \). \(^7\) The theoretical vacancy unemployment ratio \( \theta_t \) at time \( t \) implied by theory is \( \left[ \frac{\mu J_t}{k} \right]^{\frac{1}{\alpha}} \), where \( J_t \) is the PDV of corporate profits in time \( t \). Once \( \theta_t \) is computed, the implied unemployment rate is computed from the steady state accounting equation. Figure 2 below shows time-series of both data of and implied unemployment rates.


\( \alpha = 0.5 \)

\(^7\)Calibrated \( \mu \) and \( k \) are independent of choices of parameter \( \alpha \), since we normalize \( \theta = 1 \).
FIGURE 2B: Implied versus Actual Unemployment Rates in the U.S., 1960-2000
\[ \alpha = 0.3 \]

FIGURE 2C: Implied versus Actual Unemployment Rates in the U.S., 1960-2000
\[ \alpha = 0.2 \]
FIGURE 2D: Implied versus Actual Unemployment Rates in the U.S., 1960-2000
\[ \alpha = 0.1 \]

FIGURE 2E: Implied versus Actual Unemployment Rates in the U.S., 1960-2000
\[ \alpha = 0.72 \]

It is seen from Figures 2 that some, but not all of variations in the unem-
ployment rate in data is explained by the variation in corporate profits when parameter \( \alpha \) is not very small. Shimer (2005), for example, uses \( \alpha = 0.72 \) by using regression coefficients of measured job-finding rate on the measure of vacancy unemployment ratio. The simple exercise in this section offers motivations to consider an alternative specification of free entry of firms into labor markets.

3 A DMP Model with Fixed Entry Costs

In this section, I build a variant of a search and matching model of Diamond, Mortensen and Pissarides type, in which firms spend fixed costs upon entry.

3.1 Environment

Consider a continuous time economy populated by a continuum of infinitely lived risk-neutral workers and firms, both of whom discount future payoffs at rate \( r > 0 \). Individuals derive utility from consumption of their share of output. Workers and firms must match in pairs in order to produce. Each employed worker produces output \( y > 0 \) per period. A match receives an idiosyncratic separation shock at Poisson arrival rate \( s_F > 0 \).

The matching technology is described by a matching function, which gives the number of matches \( M(u,v) \), where \( u \) is the number of unemployed workers, and \( v \) the number of vacant jobs. Function \( M : \mathbb{R}^2_+ \rightarrow \mathbb{R}_+ \) is strictly increasing, strictly concave, twice-continuously differentiable, exhibits constant returns to scale, and other features described in Pissarides (2000). Then, a meeting rate for an unemployed worker is \( M(u,v)/u \equiv f(\theta) \) and the one for a vacant firm \( q(\theta) \equiv M(u,v)/v \), where \( \theta \equiv v/u \).

The value of an unemployed worker, which I denote as \( U \), satisfies

\[
ru = z + f(\theta)(W - U) \tag{1}
\]
where $z$ is a job-seeker’s flow utility while unemployed, and $W$ is the asset value of an employed worker, which must satisfy:

$$rW = w + s_F(U - W)$$

where $w$ is a flow wage of an employed worker, and $s_F$ is the arrival rate of a match separation shock. The asset value of a filled firm, employing a worker, which I denote as $J$, satisfies

$$rJ = y - w - s_FJ$$

$V$ satisfies:

$$rV = q(\theta)(J - V) - s_VV$$

where there are no flow recruiting costs while being vacant, and each vacancy receives machine destruction shocks at rate $s_V$. Each firm must incur a fixed cost $\nu_0 > 0$ when entering labor markets. Free entry into labor markets is assumed. Free entry of firms implies that

$$V = \nu_0$$

As for wage determination, a worker and employer share the match surplus in fixed proportions:

$$\frac{W - U}{\beta} = \frac{J - V}{1 - \beta}$$

where $\beta \in (0, 1)$ is workers’ bargaining power.

Steady state accounting implies that

$$u = \frac{s_F}{s_F + f(\theta)}$$

**Definition 1.** An equilibrium of the matching model with fixed entry
costs is defined as a tuple \( \{U, W, J, V, w, \theta, u\} \) such that: (i) four value functions \( U, W, J \) and \( V \) satisfy Eqs. (1), (2), (3) and (4); (ii) \( \theta \) satisfies Eq. (5); (iii) flow wage \( w \) is determined by the Nash sharing rule (6); and (iv) the unemployment rate \( u \) satisfies Eq. (7).

Sum equations (2) and (3), and then subtract equations (1) and (4) to obtain:

\[
    r(W + J - U - V) = y - z - s_F(W + J - U) + s_V V - f(\theta)(W - U) - q(\theta)(J - V)
\]

Let \( G \equiv W + J - U - V \). The Nash sharing rule (6) implies that \( W - U = \beta G \) and \( J - V = (1 - \beta)G \). Using these, we obtain:

\[
    [r + s_F + \beta f(\theta) + (1 - \beta)q(\theta)]G = y - z - (s_F - s_V)V
\]

By plugging the free entry condition \( rv_0 = q(\theta)(1 - \beta)G - s_V v_0 \) into this equation, we obtain

\[
    (r + s_V)v_0 = \frac{(1 - \beta)q(\theta)}{r + s_F + \beta f(\theta)}[y - z - (r + s_F)v_0]
\]

In what follows, I consider the following five different cases:

<table>
<thead>
<tr>
<th>TABLE 1: Five Cases Considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_F )</td>
</tr>
<tr>
<td>( s_V )</td>
</tr>
</tbody>
</table>

Notes: \( s_F \) is the rate at which a filled firm breaks, and \( s_V \) is the rate at which a vacancy breaks.

*Calibration of the DMP model with fixed entry costs under steady states.*
For \( f(\theta) \), and \( q(\theta) \), I follow the literature: \( f(\theta) = \mu \theta^{1/2} \), \( q(\theta) = \mu \theta^{-1/2} \). I normalize a time period to be one year, and therefore set the annual interest rate \( r \) to be 0.05. I let bargaining power of workers \( \beta \) to be 0.5. I normalize \( y = 1 \). I let \( z = 0.4 \). I choose the separation rate of filled jobs \( s_F = 0.4 \). I choose parameters \( \mu \) so that the unemployment rate \( u \) is 0.05. The parameter value \( \mu = 7.6 \) will do this job. I choose parameter \( \nu_0 \) so that the vacancy-unemployment ratio \( \theta \) is 1 (normalization) in equilibrium. It depends on which case to be considered. I obtain the following calibrated parameter value of \( \nu_0 \) for five different cases:

| TABLE 2: Calibrated values of entry cost \( \nu_0 \) |
|---|---|---|---|---|
| \( s_F \) | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |
| \( s_V \) | 0.4 | 0.3 | 0.2 | 0.1 | 0.0 |
| \( \nu_0 \) | 0.6294 | 0.7131 | 0.8224 | 0.9712 | 1.1860 |

TABLE3a: Productivity and the v-u ratio in steady state (\( s_V = 0.4 \))

<table>
<thead>
<tr>
<th>( y )</th>
<th>0.95</th>
<th>0.98</th>
<th>1.00</th>
<th>1.02</th>
<th>1.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>0.834</td>
<td>0.9337</td>
<td>1.00</td>
<td>1.067</td>
<td>1.167</td>
</tr>
</tbody>
</table>

Notes: \( y \) is productivity and \( \theta \) is the vacancy-unemployment ratio.

In steady states, the elasticity of the v-u ratio with respect to changes in productivity \( d \log \theta / d \log y \), corresponding to the numbers in Table 3a above, is 3.35 on average. This number is much larger than what is obtained from the standard DMP model with no fixed entry costs, but a flow vacancy costs (1.7).

| TABLE3b: Productivity and the v-u ratio in steady state (\( s_V = 0.3 \)) |
|---|---|---|---|---|
| \( y \) | 0.95 | 0.98 | 1.00 | 1.02 | 1.05 |
| \( \theta \) | 0.811 | 0.924 | 1.00 | 1.076 | 1.189 |
TABLE 3c: Productivity and the v-u ratio in steady state ($s_V = 0.2$)

<table>
<thead>
<tr>
<th>$y$</th>
<th>0.95</th>
<th>0.98</th>
<th>1.00</th>
<th>1.02</th>
<th>1.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.771</td>
<td>0.908</td>
<td>1.00</td>
<td>1.092</td>
<td>1.23</td>
</tr>
</tbody>
</table>

TABLE 3d: Productivity and the v-u ratio in steady state ($s_V = 0.1$)

<table>
<thead>
<tr>
<th>$y$</th>
<th>0.95</th>
<th>0.98</th>
<th>1.00</th>
<th>1.02</th>
<th>1.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.678</td>
<td>0.871</td>
<td>1.00</td>
<td>1.13</td>
<td>1.325</td>
</tr>
</tbody>
</table>

TABLE 3e: Productivity and the v-u ratio in steady state ($s_V = 0$)

<table>
<thead>
<tr>
<th>$y$</th>
<th>0.95</th>
<th>0.98</th>
<th>1.00</th>
<th>1.02</th>
<th>1.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.219</td>
<td>0.683</td>
<td>1.00</td>
<td>1.32</td>
<td>1.8</td>
</tr>
</tbody>
</table>

In steady states, the elasticity of the v-u ratio with respect to changes in productivity $d \log \theta/d \log y$, corresponding to the numbers in Table 3e above, is $(12+30+19+14)/4 = 18.7$ on average. This number is much larger than what is obtained from the standard DMP model with no fixed entry costs, but a flow vacancy costs (1.7). Moreover, this number is slightly larger than data. It may be safe to say that the size of the impact of productivity changes on the model-generated vacancy unemployment ratio is in the empirically sensible ranges. The reason why the DMP model with fixed entry costs delivers such large impacts of productivity changes on the v-u ratio is seen from equation (8). In calibration, $y = 1$, and $z + (r + s_F)\nu_0 = 0.9337$. This is compared with the calibration of Hagedorn and Manovskii (2006), in which they choose $z = 0.9$, and there is no entry costs in their model ($\nu_0 = 0$).
4 A DMP Model with Flow Recruiting Costs

One modification is to replace an equation for an unfilled vacancy’s asset value and a free entry equation by the following free entry condition under the setup of flow vacancy costs:

\[ \frac{k}{q(\theta)} = J \]  \hspace{1cm} (9)

where \( k > 0 \) is a flow vacancy cost. The second modification is to replace the Nash bargaining equation by:

\[ \frac{W - U}{\beta} = J \frac{1}{1 - \beta} \]  \hspace{1cm} (10)

With these two modifications, a steady state equilibrium for the case of flow vacancy costs is analogously defined as the case of fixed entry costs examined above.

**Definition 2.** An equilibrium of the matching model with flow vacancy costs is defined as a tuple \( \{U, W, J, w, \theta, u\} \) such that: (i) three value functions \( U, W, J \) satisfy Eqs. (1), (2), and (3); (ii) \( \theta \) satisfies the free entry Eq. (9); (iii) flow wage \( w \) is determined by the Nash sharing rule (10); and (iv) the unemployment rate \( u \) satisfies Eq. (7).

Then, one can show that a steady state equilibrium is characterized by \( \{\theta, u\} \) satisfying the following two equations.

\[ k = \frac{q(\theta)(1 - \beta)}{r + s_F + \beta f(\theta)} (y - z) \]  \hspace{1cm} (11)

and Eq. (7). This is seen as follows. First, sum equations (2) and (3) and then subtract equation (1), we obtain

\[ rG = y - z - f(\theta)(W - U) - s_F G \]

where \( G \equiv W + J - U \). The Nash sharing rule (10) implies that \( W - U = \beta G \).
and $J = (1 - \beta)G$. Substituting for $W - U$ in the equation for $G$, we obtain:

$$rG = y - [z + f(\theta)\beta] - sF G$$

Free entry equation is written as $k = q(\theta)(1 - \beta)G$. Plugging this into the expression for $G$, we obtain equation (11) above.

**Calibration of the DMP model with flow vacancy costs (standard) under steady state.**

Under the same parameter values used above, the flow vacancy cost parameter $k$ is calibrated as $k = 0.5365$.

<table>
<thead>
<tr>
<th>TABLE 4: Productivity and the v-u ratio in steady state (A model with flow vacancy costs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
</tr>
<tr>
<td>$\theta$</td>
</tr>
<tr>
<td>$u$</td>
</tr>
</tbody>
</table>

In steady state, the elasticity of the v-u ratio with respect to changes in productivity $d \log \theta / d \log y$, corresponding to the numbers in Table 4 above, is 1.76 on average.

## 5 Extension: Aggregate Shocks

I build and calibrate a variant of a search matching model with two-state productivity shocks. There are two aggregate states $p \in \{B, G\}$. Productivity is high ($y_G$) in state G, and low ($y_B$) in state B., where $y_G > y_B$. Aggregate state $p$ transits its state to a new state at rate $\pi_p$ for $p \in \{B, G\}$. $\pi_B$ is the exit rate from state B and $\pi_G$ exit rate from state G. My finding in this section is summarized by the following table:
### TABLE 5: Calibration under Three Different Setups

<table>
<thead>
<tr>
<th></th>
<th>$\theta_G$</th>
<th>$\theta_B$</th>
<th>Coef. Var ($\theta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark (flow vacancy costs)</td>
<td>1 (normalization)</td>
<td>0.9428</td>
<td>0.0205</td>
</tr>
<tr>
<td>Fixed entry cost, $s_V = 0.4$</td>
<td>1 (normalization)</td>
<td>0.8891</td>
<td>0.0402</td>
</tr>
<tr>
<td>Fixed entry cost, $s_V = 0$</td>
<td>1 (normalization)</td>
<td>0.4854</td>
<td>0.1986</td>
</tr>
</tbody>
</table>

Notes: Parameter $k$ or $\nu_0$ is chosen so that each model delivers $\theta_G = 1$ as a normalization. See the main text for details.

### 5.1 A DMP with Flow Vacancy Costs

The value of unemployment in state $p$, which is denoted by $U_p$, satisfies:

$$rU_p = z + f(\theta_p)[W_p - U_p] + \pi_p[U_{-p} - U_p] \quad (12)$$

where $r$ is the interest rate, $z$ is a flow leisure value, $f(\theta_p)$ is the rate at which a job seeker contacts a vacant job, $W_p$ is the employment value for workers in state $p$, $\tilde{p}$ denotes the other state. The value of employment for workers in state $p$, $(W_p)$, satisfies:

$$rW_p = w_p + s(U_p - W_p) + \pi_p[W_{-p} - W_p] \quad (13)$$

where $w_p$ is a wage per period in state $p$, $s$ is the exogenous match separation rate. Free entry of vacant firms implies that:

$$\frac{k}{q(\theta_p)} = J_p \quad (14)$$

where $k > 0$ is a flow recruiting cost incurred by vacant firms, $q(\theta_p)$ is the rate at which each vacancy contacts a job seeker when the vacancy unemployment ratio is $\theta_p$, and $J_p$ is the value of filled firms in state $p$. The value of filled firm at state $p$ ($J_p$) satisfies:

$$rJ_p = y_p - w_p - sJ_p + \pi_p(J_{-p} - J_p) \quad (15)$$
where $y_p$ is employees’ flow productivity in state $p$. Wage $w_p$ in state $p$ is determined by a sharing rule implies by Nash Bargaining:

$$\frac{W_p - U_p}{\beta} = \frac{J_p}{1 - \beta}$$

(16)

The steady state accounting implies that:

$$u_p = \frac{s}{s + f(\theta_p)}$$

(17)

An equilibrium for the search matching model with flow recruiting costs described above is a list of \{U_p, W_p, J_p, \theta_p, w_p, u_p\} for $p \in \{G, B\}$ such that: \{U_p, W_p, J_p\} satisfy equations (12), (13) and (15); \theta_p satisfies equation (14); $w_p$ satisfies equation (16); and $u_p$ satisfies equation (17). One can obtain the following non-linear equations in $\theta_p$.

$$\frac{r + s + \pi_p}{q(\theta_p)} + \beta \theta_p = (1 - \beta) \frac{y_p - z}{k} + \pi_p \frac{1}{q(\theta_p)}$$

(18)

This equation is derived as follows. Sum equations (13) and (15), and then subtract equation (12) to obtain:

$$r(W_p + J_p - U_p) = y_p - z - s(W_p + J_p - U_p) - f(\theta_p)(W_p - U_p)
+ \pi_p[(W_p + J_p - U_p) - (W_p + J_p - U_p)]$$

Let $G_p \equiv W_p + J_p - U_p$. The Nash sharing rule (16) implies that $W_p - U_p = \beta G_p$ and $J_p = (1 - \beta)G_p$. Then, we obtain:

$$(r + s + \pi_p)G_p = y_p - z - f(\theta_p)\beta G_p + \pi_p G_p$$

By plugging the free entry equation $k = q(\theta_p)(1 - \beta)G_p$ into the equation just above, we obtain equation (18).

Calibration of the DMP model with flow vacancy costs (standard) under
two aggregate state economy.

Let \( \pi_G = 12/57 \) and \( \pi_B = 12/10 \). These two numbers are taken from the U.S. business cycle experiences in which duration of an expansion is 57 months and recessions 10 months. Let a matching function be \( q(\theta) = \mu \theta^{-0.5} \). I choose a model’s one unit of time to be one year: \( r = 0.05 \), \( s = 0.42 \), and \( z = 0.4 \). I normalize \( y_B = 1 \). I pick \( y_G \) so that the coefficient of variation of productivity is 0.014. \( y_G = 1.042 \) will do. I also normalize \( \theta_G = 1 \), which is equivalent to letting \( k = 0.5732 \). Then, this model delivers \( \theta_B = 0.9428 \). The coefficient of variation in the v-u ratios in this economy is computed as 0.0205, which is about 1/10 of data.

5.2 A DMP with Fixed Entry Costs: Both Vacancies and Filled Firms Break at the Same Rate

The asset value of a vacant job in state \( p \) satisfies:

\[
rV_p = q(\theta_p)(J_p - V_p) - sV_p + \pi_p(V_p - V_p)
\]  

(19)

The asset value of a filled job in state \( p \) satisfies:

\[
rJ_p = y_p - w_p - sJ_p + \pi_p(J_p - J_p)
\]  

(20)

Free entry of firms implies that:

\[ V_p = \nu_0 \]  

(21)

The Nash bargaining implies that:

\[
\frac{W_p - U_p}{\beta} = \frac{J_p - V_p}{1 - \beta}
\]  

(22)

An equilibrium for the search matching model with fixed entry costs, in which both vacancies and filled firms break at the same rate, described above is a list of \( \{U_p, W_p, J_p, V_p, \theta_p, w_p, u_p\} \) for \( p \in \{G, B\} \) such that:
\{U_p, W_p, J_p, V_p\} satisfy equations (12), (13), (19) and (20); \(\theta_p\) satisfies equation (21); \(w_p\) satisfies equation (22); and \(u_p\) satisfies equation (17).

In this case, the equation for \(\theta_p\) is computed as:

\[
\frac{r + s + \pi_p}{q(\theta_p)} + \beta \theta_p = (1 - \beta) \frac{y_p - z - r\nu_0}{r\nu_0} + \pi_p \frac{1}{q(\theta_p)} \tag{23}
\]

This equation is obtained as follows. Let \(G_p \equiv W_p + J_p - U_p - V_p\). Sum equations (13) and (20), and then subtract equations (12) and (19) to obtain:

\[
rg_p = yp - z - sG_p - f(\theta_p)\beta G_p - q(\theta_p)(1 - \beta)G_p \\
+ \pi_p(G_p - G_p)
\]

where I used the implication of the Nash sharing rule: \(W_p - U_p = \beta G_p\) and \(J_p - V_p = (1 - \beta)G_p\). The equation just above is simplified to:

\[
[r + s + \beta f(\theta_p) + (1 - \beta)q(\theta_p)]G_p = yp - z + \pi_p(G_p - G_p)
\]

Plug the free entry condition \(r\nu_0 = q(\theta_p)(1 - \beta)G_p\) into the equation just above. Then, we obtain equation (23).

**Calibration of the DMP model with fixed entry costs, in which both vacancies and filled firms break at the same rate.**

Let \(\pi_G = 12/57\) and \(\pi_B = 12/10\). These two numbers are taken from the U.S. business cycle experiences in which duration of an expansion is 57 months and recessions 10 months. Let a matching function be \(q(\theta) = \mu \theta^{-0.5}\). I choose a model’s one unit of time to be one year: \(r = 0.05\), \(s = 0.42\), and \(z = 0.4\). I normalize \(y_B = 1\). I pick \(y_G\) so that the coefficient of variation of productivity is 0.014. \(y_G = 1.042\) will do. I also normalize \(\theta_G = 1\), which is equivalent to letting \(\nu_0 = 6.0521\). Then, this model delivers \(\theta_B = 0.8891\). The coefficient of variation in the v-u ratios in this economy is computed as 0.0402, which is about twice the benchmark case under flow vacancy costs.
5.3 A DMP with Fixed Entry Costs: Vacancies do not break

The only modification to the model with fixed entry costs, in which vacancies are with machines, is to replace equation (19) by the following equation:

\[ rV_p = q(\theta_p)(J_p - V_p) + \pi_p(V_p - V) \]  

(24)

The equation for \( \theta_p \) is computed as

\[
\frac{r + s + \pi_p}{q(\theta_p)} + \beta \theta_p = \left(1 - \beta\right) \frac{y_p - z - (r + s)\nu_0}{r\nu_0} + \pi_p \frac{1}{q(\theta_p)}
\]  

(25)

This equation is obtained as follows. Let \( G_p \equiv W_p + J_p - U_p - V_p \). Sum equations (13) and (15), and then subtract equations (12) and (24) to obtain:

\[
rG_p = y_p - z - sG_p - sV_p - f(\theta_p)\beta G_p - q(\theta_p)(1 - \beta)G_p + \pi_p(G_p - G_p)
\]

where I used the implication of the Nash sharing rule: \( W_p - U_p = \beta G_p \) and \( J_p - V_p = (1 - \beta)G_p \). The equation just above is simplified to:

\[
[r + s + \beta f(\theta_p) + (1 - \beta)q(\theta_p)]G_p = y_p - z - sV_p + \pi_p(G_p - G_p)
\]

Plug the free entry condition \( r\nu_0 = q(\theta_p)(1 - \beta)G_p \) into the equation just above. Then, we obtain equation (25).

**Calibration of the DMP model with fixed entry costs, in which vacancies do not break.**

Let \( \pi_G = 12/57 \) and \( \pi_B = 12/10 \). These two numbers are taken from the U.S. business cycle experiences in which duration of an expansion is 57 months and recessions 10 months. Let a matching function be \( q(\theta) = \mu \theta^{-0.5} \). I choose a model’s one unit of time to be one year: \( r = 0.05, s = 0.42, \) and
z = 0.4. I normalize $y_B = 1$. I pick $y_C$ so that the coefficient of variation of productivity is 0.014. $y_C = 1.042$ will do. I also normalize $\theta_C = 1$, which is equivalent to letting $\nu_0 = 1.266869$. Then, this model delivers $\theta_B = 0.4854$. The coefficient of variation in the v-u ratios in this economy is computed as 0.1986, which is about the same as data!

6 Why Entry Costs? A Model with Schooling Choices

The finding in previous sections raises the following two important questions: (i) Which specification of entrepreneurs’ costs before production is more plausible?; and (ii) What is the sensible value of $s_V$, the rate at which a vacant position disappears?. I stress that answering these questions is important to make my argument of introducing entry costs more convincing. Since I am aware of no direct evidence of entrepreneurs’ entry costs and vacant positions’ destruction rate, my strategy here is to enrich the model slightly so that it delivers different kinds of predictions, which can be compared with data. To do so, in this section, I add choices of time-consuming schooling into otherwise standard matching models of unemployment. Time-consuming schooling is necessarily non-decreasing in productivity levels when firms incur only flow recruiting costs while being vacant. However, it can be either increasing or decreasing in productivity levels when firms incur fixed entry costs. Since the unemployment rate is decreasing in worker productivity levels, this finding suggests that the model with flow vacancy costs and no entry costs predicts the weakly negative relation between the unemployment rate and school attendance: weakly procyclical school attendance. However, a number of studies suggests that school attendance at the level of community college level is strongly countercyclical (Betts and McFarland 1995; Dellas and Sakellaris 2003; DeJong and Ingram 2001). The model with entry costs is consistent with this exiting empirical findings, since it predicts
either positive or negative relations between the unemployment rate and the school attendance. The empirical studies cited above also provides disciplines in choosing the plausible value of $s_V$, the rate at which each vacant position disappears.

## 6.1 Model with Fixed Entry Costs

Consider a continuous time economy populated by a continuum of infinitely lived risk-neutral workers and firms, both of whom discount future payoffs at rate $r > 0$. Individuals derive utility from consumption of their share of output. A newly-born worker chooses the level of human capital $h \in \mathbb{R}_+$ before entering labor markets as an unemployed worker. It takes $h \geq 0$ periods to acquire human capital $h$ at school. Let $c(h)$ denote the utility cost to a worker from choosing schooling $h$. Function $c(h)$ is twice-continuously differentiable, where $c'(h) \geq 0, c''(h) \geq 0$, and $\lim_{h \to 0} c'(h) = 0$. Workers and firms must match in pairs in order to produce. Each employed worker, who has schooling $h$, produces output $Ay(h)$ per period, where $y(h)$ is strictly increasing in $h$, and parameter $A > 0$ captures aggregate productivity. A match receives an idiosyncratic separation shock at Poisson arrival rate $s > 0$. The matching technology is the same as the previous sections.

Only steady-states are considered in what follows. The value of an unemployed worker, who has invested in schooling $h$, which I denote as $U(h)$, satisfies

$$rU(h) = f(\theta)[W(h) - U(h)]$$

(26)

where $W(h)$ is the asset value of an employed worker who chooses schooling $h$, which must satisfy:

$$rW(h) = w(h) + s[U(h) - W(h)]$$

(27)

where $w(h)$ is a flow wage of an employed worker with schooling $h$, and $s$
is the arrival rate of a match separation shock. The asset value of a filled firm, employing a worker with the level of human capital $h$, which I denote as $J(h)$, satisfies

$$ rJ(h) = Ay(h) - w(h) - sJ(h) \quad (28) $$

The asset value of an unfilled vacant firm, who conjectures that an equilibrium schooling is $H$, satisfies:

$$ rV(H) = q(\theta)[J(H) - V(H)] \quad (29) $$

where there are no flow recruiting costs while being vacant. Each firm must incur a fixed cost $\nu_0 > 0$ when entering labor markets. Free entry into labor markets is assumed. Free entry of firms implies that

$$ V(H) = \nu_0 \quad (30) $$

As for wage determination, a worker and employer share the match surplus in fixed proportions:

$$ \frac{W(h) - U(h)}{\beta} = \frac{J(h) - V}{1 - \beta} \quad (31) $$

where $\beta \in (0, 1)$ is workers’ bargaining power. Steady state accounting is given by equation (7).

**Definition 3.** An equilibrium is defined as a tuple \{ $U(h)$, $W(h)$, $J(h)$, $V(H)$, $w(h)$, $\theta$, $h$, $H$, $u$ \} such that: (i) four value functions $U(h)$, $W(h)$, $J(h)$, and $V(H)$ satisfy Eqs. (26), (27), (28) and (29); (ii) $\theta$ satisfies Eq. (30), given $H$; (iii) flow wage $w(h)$ is determined by the Nash sharing rule (31); (iv) individual schooling $h$ maximizes $\exp(-rh')U(h') - c(h')$ over $h'$ given $H$; (v) rational expectation consistency $h = H$; and (vi) the unemployment rate $u$ satisfies Eq. (7).

The value of unemployment for workers with individual human capital $h$
is computed as \( rU(h) = \Pi(\theta)[y(h) - (r + s)\nu_0] \) where

\[
\Pi(\theta) \equiv \frac{\beta f(\theta)}{r + s + \beta f(\theta)} \tag{32}
\]

With some algebra, a steady state equilibrium is characterized by a pair \((h, \theta)\) satisfying the following two equations:

\[
\begin{align*}
\frac{Ay'(h)}{r} &= [Ay(h) - (r + s)\nu_0] + \frac{c'(h)\exp(rh)}{\Pi(\theta)} \tag{33} \\
r\nu_0 &= \frac{1 - \beta}{\beta} \frac{\Pi(\theta)}{\theta}[Ay(h) - (r + s)\nu_0] \tag{34}
\end{align*}
\]

The case of instant schooling is examined by Laing et al. (1995).

**Comparative Statics.**

Let \( \Psi(h) \equiv \frac{Ay'(h)}{r} - [Ay(h) - (r + s)\nu_0] - \frac{c'(h)\exp(rh)}{\Pi(\theta)} \), where \( \Theta(h) \) is defined by \( r\nu_0 = \frac{1 - \beta}{\beta} \frac{\Pi(\Theta(h))}{\Theta(h)}[Ay(h) - (r + s)\nu_0] \).

**Proposition 4.** Consider the search matching model with fixed entry costs of vacancies. Then, we have:

(I) Assume that \( c'(h) = 0 \) for all \( h \geq 0 \). Then, \( \frac{dh}{\nu_0} > 0 \) and \( \frac{dh}{A} < 0 \).

(II) Assume that \( c'(h) > 0 \) for all \( h \geq 0 \), and \( \Psi(h) \) is decreasing in \( h \) for all \( h \). Then we have: \( \frac{dh}{\nu_0} \) and \( \frac{dh}{A} \).

**Proof.** Consider Case I first. When \( c'(h) = 0 \) \( \forall h \), an equilibrium pair \((\theta, h)\) is given by Eq. (34) and the following human capital Eq. \( \frac{Ay'(h)}{r} = Ay(h) - (r + s)\nu_0 \). The result follows directly from this equation. Consider, next, Case II. An equilibrium \( h \) is given by \( \Psi(h) = 0 \), where \( \Psi(h) \) is decreasing in \( h \). Once \( h \) is determined, equilibrium \( \theta \) is determined by \( \Theta(h) \). An increase in \( \nu_0 \) increases the second term and reduces the third term in \( \Psi(h) \). An increase in \( A \) increases the first and the third term of \( \Psi(h) \), but decreases the second term. Analytically, the effect of changes in either \( \nu_0 \) or \( A \) on equilibrium \( h \) is ambiguous.
The assumption that $\Psi'(h) < 0$ for all $h$ in Case II is imposed to focus on the plausible case in which an increase in marginal monetary costs of education $c'(h)$ reduces schooling in equilibrium. Part 1 of Proposition 4 suggests that when marginal non-time costs of schooling, such as tuition and mental scholarly effort costs, is zero, an increase in productivity reduces schooling. Since an increase in productivity reduces the unemployment rate, this suggests that one should observe the positive relation between the school attendance and the unemployment rate. This is consistent with a number of studies cited earlier (Betts and McFarland; Dellas and Sakellaris; and DeJong and Ingram). Part 2 of Proposition 4 says that when the marginal costs of tuition and scholarly effort is positive, an increase in productivity either increases or decreases schooling levels. The first order condition says that the opportunity cost effect tends to be dominated as the size of $c'(h)$ becomes larger. As the size of tuition and effort costs becomes larger, the more likely it is that an increase in productivity increases schooling levels. This prediction is consistent with three empirical studies mentioned above and Manski and Wise (1983) who show that enrollments in four-year college is not significantly related with the unemployment rate, given that tuition and effort costs are much larger in four-year colleges than for two-year community colleges, which appear empirically plausible.

6.2 Model with Flow Vacancy Costs

Next, the standard case, in which vacancies incur flow vacancy costs and firms do not incur fixed entry costs, is examined. One modification is to replace an equation for an unfilled vacancy’s asset value and a free entry equation by the following free entry condition under the setup of flow vacancy costs:

$$\frac{k}{q(\theta)} = J(H)$$

(35)

where $k > 0$ is a flow vacancy cost. The second modification is to replace the Nash bargaining equation by:
With these two modifications, a steady state equilibrium for the case of flow vacancy costs is analogously defined as the case of fixed entry costs examined above. Then, one can show that a steady state equilibrium is characterized by \( \{\theta, h, u\} \) satisfying the following three equations.

\[
W(h) - U(h) = \frac{J(h)}{1 - \beta}
\]

(36)

and Eq. (7). Let \( \theta \) satisfying Eq. (37) given \( h \) be denoted by \( \Theta(h) \), and let \( \Psi(h) \) be defined by

\[
\Psi(h) = Ay'(h) - rAy(h) - \frac{rc'(h)e^{rh}}{\Pi(\theta)}
\]

(38)

\( k = \frac{1 - \beta \Pi(\theta)}{\beta} Ay(h) \)

(37)

Proposition 5. Consider the search matching model with flow costs of vacancies. Then, we have:

(I) Assume that \( c'(h) = 0 \) for all \( h \geq 0 \). Then, \( \frac{dh}{dk} = 0 \) and \( \frac{dh}{dA} = 0 \).

(II) Assume that \( c'(h) > 0 \) for all \( h \geq 0 \), and \( \Psi(h) \) is decreasing in \( h \) for all \( h \). Then we have: \( \frac{dh}{dk} < 0 \) and \( \frac{dh}{dA} > 0 \).

Proof. First, consider the case in which \( c'(h) = 0 \) for all \( h \geq 0 \). Equilibrium schooling \( h_e \) is given by \( y'(h_e) = ry(h_e) \). The result follows from this equation. Next, consider the case in which \( c'(h) \neq 0 \) for all \( h \geq 0 \). Equilibrium education \( h_e \) is given by \( 0 = \Psi(h_e) \). An increase in vacancy costs \( k \) reduces \( \Theta(h) \) for given levels of \( h \), which reduces \( \Psi(h) \) for given levels of \( h \). This unambiguously reduces schooling in equilibrium. The effect of changes in parameter \( A \) is examined in the similar way. ||

The assumption that \( \Psi(h) \) is decreasing in \( h \) for all \( h \) is made in order to
focus on the plausible case in which an increase in $c'(h)$ reduces schooling. Proposition 5 suggests that in the economy with no entry costs and only flow recruiting costs, an increase in worker productivity weakly increases schooling: the negative relation between the school attendance and the unemployment rate, regardless of the size of tuition and scholarly effort costs. This strong prediction is not in accordance with the empirical studies mentioned above.

I conclude, from Propositions 4 and 5 alike, that the matching models with entry costs are more empirically plausible than the ones with no entry costs and only flow recruiting costs. This provides strong motivations to recalibrate a variant of matching models with entry costs and examine its quantitative impacts of productivity changes on vacancies and unemployment, which is what I did in Section 3 in the present paper. I found in Section 3 that the matching model with entry costs can deliver the larger impacts of productivity changes on the vacancy unemployment ratio than the models with no entry costs and only flow vacancy costs.

**Calibration.**

In the appendix, the matching model with time-consuming schooling choices and fixed entry costs are calibrated using data of the U.S. labor markets and educational attainments. An increase in productivity increases schooling years slightly and an increase in entry costs also increases schooling slightly under such calibrated economy. This framework can be a basis for choosing the appropriate value of parameter $s_V$, the rate at which vacant jobs disappear.  

89 This work is currently in progress.

7 Conclusion

This paper has assessed the matching models of unemployment with two different setups: one with no entry costs and flow vacancy costs as is common
in the literature; and the other with entry costs and no flow vacancy costs. The model with the first setup (only vacancy costs) has two kinds of difficulties. First, it cannot deliver the impact of productivity changes on the vacancies and unemployment of an empirical size, which is found by Hall and Shimer. Second, it cannot produce the empirically observed positive relation between the college enrollments and the unemployment rate. This is my finding in the present paper. I demonstrated that the model with the second setup (with entry costs) can deliver the positive relation between the college enrollments and the unemployment, provided that the tuition and scholarly effort costs are not very large. I interpret this finding as motives for examining the matching models with the second setup (entry costs). I found that such modes with entry costs can deliver the large impacts of productivity changes on vacancies and unemployment, whose magnitude is not far from the U.S. data.

Appendix.

In this appendix, I calibrate the matching model with time-consuming schooling and fixed entry costs, examined in Section 6, to see whether an increase in either fixed entry costs or aggregate productivity increases schooling in equilibrium, when the model is calibrated to data of the U.S. labor markets and educational attainments.

Assume that students make decisions on their years of schooling at the time of completing the 9th grade. According to the CPS, the average fraction of each educational category in the civilian labor force during the periods from December 1999 through November 2000 is as follows: Less High School (0.102); High School (0.315); Some College (0.278); and College Plus (0.310). The average of schooling in years (after the 9th grades) weighted by the fraction of the civilian labor force by educational attainment is 4.6 yrs. ³

For \(y(h)\), I let \(y(h) = A \exp[\phi(h + 9)]\), where \(\phi > 0\) is a private return to

³I adopt the following scaling of years of schooling: Some High School (1 year); High School (3 years); Some College (5 years); and College Plus (7 years).
education.¹⁰ I choose $\phi = 0.1$. For $c(h)$, I let $c(h) = c_0 \exp(\eta h)$. For $p(\theta)$, and $q(\theta)$, I follow the literature: $p(\theta) = \mu \theta^{1/2}$, $q(\theta) = \mu \theta^{-1/2}$.

The parametrized equations determining $h$, $\theta$, and $u$ in equilibrium are given by:

\begin{align*}
\phi \exp[\phi(h+9)] &= r[\exp\{\phi(h+9)\} - (r + s)\frac{\nu_0}{A}] + \frac{c_0 \eta \exp\{(r + \eta)h\}}{\Pi(\theta)} \\
\exp\{\phi(h+9)\} &= r \frac{\nu_0}{A} \left[ \frac{r + s}{r} + \frac{\beta}{1 - \beta} \frac{\theta}{\Pi(\theta)} \right] \\
\exp\{\phi(h+9)\} &= s \frac{s}{s + \mu \theta^{0.5}}
\end{align*}

The following second order condition for choices of schooling should be checked:

$$\phi^2 - r \phi - \frac{c_0}{A} \frac{\eta}{\Pi(\theta)} (r + \eta) \exp\{(r + \eta - \phi)h\} < 0$$

I normalize a time period to be one year, and therefore set the annual interest rate $r$ to be 0.05. I let bargaining power of workers $\beta$ to be 0.5. I choose the separation rate $s = 0.4$. I choose the value of $\eta = 0.76$ from the tuition data for each education level.¹¹

¹⁰This specification is inspired by the form of Mincerian equations. Observe that I adjust the scaling of years of schooling, since the coefficient $\phi$ is taken from the estimated Mincerian equation, where the scaling of years of schooling is such that high school diplomas ($h=12$), bachelors degree ($h=16$) and so on. Bils and Klenow (2000), for instance, reports that:

$$\log w = \text{const} + 0.093s + 0.032(age - s - 6) - 0.00048(age - s - 6)^2$$

where $w$ is the level of earnings in 1989 in USA, $s$ is years of schooling, and $age - s - 6$ captures an experience. By focusing on workers with no experience, I choose $\phi = 0.1$.

¹¹To obtain the value of $\eta$, we collect data of the tuition costs for different education categories from *Digest of Education Statistics 2001* (U.S. Department of Education). The Table 316 of the *Digest* says that the average undergraduate tuitions (annual) in 1999-2000 are $7,044 for 4 year institutions and $1,721 for 2 year institutions. The table 319 of the *Digest* says that the average graduate and first professional tuition for 1999-2000
I choose three parameters $\mu, \nu_0/A$, and $c_0/A$. To do that, I use the following three observations: schooling in years $h$ is 4.6, the unemployment rate $u$ is 0.05, and the vacancy-unemployment ratio $\theta$ is 1 (normalization). Then, I obtain the following calibrated parameters:

$$\mu = 7.6, \quad \frac{\nu_0}{A} = 7.7012, \quad \frac{c_0}{A} = 0.0104$$

I checked that the second order condition for schooling choices is satisfied in equilibrium.

We obtain the following results of comparative statics using the calibrated model. Suppose that an entry cost ($\nu_0$) increases by one percent. Then a new equilibrium will have that $\theta = 0.91$, $h = 4.6047$, and $u = 0.0523$. Under this calibrated economy with time-consuming schooling and fixed entry costs, a one percent increase in entry costs increases schooling. This is in contrast to one comparative statics result in *instant* schooling economies: an increase in entry costs reduces schooling (Laing et al.). Next, consider a one percent increases in the TFP level ($A$). A new equilibrium will have that $\theta = 1.11$, $h = 4.6148$, and $u = 0.0476$. Under the economy with time-consuming schooling and fixed entry costs, a small increase in aggregate productivity increases schooling.

**References.**

is $8,062. The table 61 of the *Digest* tells that the average tuition paid by students at private secondary schools for 1993-1994 is $4,578. Table 56 of the *Digest* says that the enrollment sizes of grades 9-12 in Fall 1999 are 13,369 for public schools and 1,254 for private schools (unit: thousands). Assuming that a tuition cost of public secondary schools is zero, the average tuition cost for secondary school is about 400. At the time of completing the 9th grades, the expected present discounted value of the tuition cost of 3-year high school educations is about 1,144. Similarly, at the time of completing the 9th grades, the expected present discounted value of costs of completing 4-year college education is about 1,144+22,655. I compute $\eta$ by:

$$\eta = \frac{\log(1144 + 22655) - \log(1144)}{4 - 0} \approx 0.76$$


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