ECONOMIC GROWTH AND CLIMATE CHANGE WITHOUT AN ENVIRONMENTAL TREATY

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The objective of this paper is to view climate change in the framework of economic growth under no controls on emissions as the baseline modeling for further research in formulating an environmental treaty. This paper analyses the economic growth with dynamic of emissions and its impacts under an overlapping-generation model. The burning of fossil fuels - main source of greenhouse gases, and renewable energy allowing a sustainable development are introduced into the model. Moreover, this paper contains explicit formalizations of the depletion process of exhaustible fossil fuels and the phase of technology substitution. There exists a competitive equilibrium under which the world's stock of fossil fuels is depleted in finite time, and depending on the values of the parameters, there might also exist an inefficient competitive equilibrium under which part of the world's stock of fossil fuels is left in situ forever unexploited. The paper also studies the impact of climate change on capital flows and welfare across countries.

1. INTRODUCTION

It has been warned that the accumulation of greenhouse gases is likely to lead to global warming and other significant climatic changes. Climate change is global in its causes and consequences, and currently a serious environmental threat. Therefore, the actions on climate change require a deep international co-operation and the economic analysis must be global and deal with long time horizons. To stabilize the greenhouse gas concentration in the atmosphere, the United Nations Framework Convention on Climate Change has set out the objective of reducing global greenhouse gas emissions to a certain level. The Stern Review (2007), the largest and most widely known report, indicate that one percent of global GDP per annum is required to be invested in order to avoid the worst effects of climate change. To that end, as of June 2007, 172 countries have ratified the Kyoto Protocol. However, ratification does not imply a country has agreed to cap their emissions. China and India - the major polluters in the world - maintain that the major responsibility of curbing emission rests with the developed countries. The United States also refuse to take actions globally. One of the most controversial issues about the Kyoto Protocol is that it is based on equity rather on efficiency grounds. Nordhaus and Yang (1996, NY hereafter); Nordhaus and Boyer (2000) discuss this issue in the RICE (1996) and the revised RICE (2000) models built to study the environmental and economic impacts of alternative approaches to climate change policy. NY show that, at the present time, it is this non-cooperative strategy adopted by most nations. In addition, in the absence of a world government, an effective agreement to control emissions should be

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self-enforcing – i.e., every country will be in its self-interest to abide by the treaty. A difficulty with the Kyoto Protocol is that it does not appear to lay the groundwork for a self-enforcing treaty which is discussed by Finus (2001), Barrett (2003), Dutta and Radner (2004). The objective of this paper is to view climate change in the framework of economic growth theory in which there are no controls on the emissions of greenhouse gases as the baseline modeling strategy for further research in formulating a self-enforcing environmental treaty.

Most existing literature is a static analysis making the assumption that GHGs decay within a single period of the model. Moreover, many papers studied the problem faced by a central planner who maximizes the utility of a representative agent, but left the inter-generational discussion untouched. This paper is a generalization of a multi-country world adopting the standard approach of modern optimal economic growth theory and incorporating both the dynamics of emissions and its impacts into an overlapping-generations framework in which an individual agent has a two-period life caring for her young and old age consumptions, and transfers her wealth over time through either oil or capital asset, or both. This paper focuses on environmental externalities from production. The new features of our model are introducing backstop energy input into the production allowing the economy to evolve along a path of sustainable development; and modeling the burning of fossil fuels. This study shows that there exists a competitive equilibrium under which the world's stock of fossil fuels is depleted in finite time. Furthermore, depending on the values of the parameters, there might also exist a competitive equilibrium under which part of the world's stock of fossil fuels is left in situ forever unexploited. However, the equilibrium under incomplete oil exhaustion is inefficient. Another interesting result is that if the world's initial stock of fossil fuels is large, then the initial global demand for oil will be high, and the backstop sector in each country will not be active in period 0, however, it will be brought into use in finite time. In light of these results, a competitive equilibrium as consisting of three phases is characterized. In the first phase, fossil fuels provide all the energy needs of the world economy. During this phase, the price of oil rises steadily at the rate of interest. The second phase – which might or might not exist – begins when the price of oil has risen to the level of the interest rate. In this phase, the two technologies – fossil fuels and backstop – co-exist, and the energy inputs used in the production of the consumption good consist of both oil and renewable energy. In the third phase of the competitive equilibrium – the post fossil fuel phase – the backstop completely takes over and provides all the energy needs of the world economy.

This paper also investigates the economic impacts of climate change in different countries. The findings are that for each generation that is born after the world's stock of fossil fuels has been depleted, the welfare of each of its members varies across countries inversely with the negative impact that climate change on its own country of origin, and in any period after the world's stock of fossil fuels has been depleted, a country that is severely affected by climate change will import capital, while a country that is impervious to climate change will export capital.

The paper is organized as follows. Section 2 presents the model. The existence of competitive equilibrium for an economy with fossil fuels is discussed in Section 3. Section 4 presents the temporary equilibria. The characterization of the competitive equilibria is analyzed in Section 5. Numerical examples to illustrate these results are presented in Section 6. Section 7 contains some concluding remarks.

2. THE MODEL

Time is discrete and denoted by $t, t = 0,1,...$ Countries are index by $i, i \in I$, with I representing the set of countries. In each period, four classes of economic agents co-exist in each country: a young generation, an old generation, competitive firms producing a consumption good, and competitive firms producing renewable energy. The consumption good is produced using capital, labor, and energy. We shall assume that the consumption good can also be used as investment goods to accumulate capital. The energy inputs come from two sources: fossil fuels, say oil, and a backstop. Furthermore, the renewable energy produced by the backstop also uses capital, say solar collectors.

The consumption good, oil, and renewable energy are freely traded. Capital is also perfectly mobile between sectors and across countries, so that at any instant there is a single interest rate on the global capital market. An individual lives two periods, working when she is young and retiring when she is old. In each period the real assets that belong to a country are owned by the old generation of that country. A young individual in a country in any period owns nothing except for a unit of time that she supplies in-elastically in the labor market of her country of origin. Part of her wage is spent on current consumption; the remaining part is saved to provide for her old-age consumption.

In any period, the state of country i is represented by the list $(X_{i,t}, K_{i,t}, N^0_{i,t}, N^1_{i,t})$, where $X_{i,t} \geq 0$, $K_{i,t}$, $N^0_{i,t}$, and $N^1_{i,t}$ represent, respectively, its stock of oil, the capital stock it owns, the number of young individuals, and the number of old individuals. The population in each country is assumed to grow at the constant rate $n \geq 0$ per period, so that $N^0_{i,t+1} = (1+n)N^0_{i,t}, t = 0,1,...$. We assume that $(X_{i,0}, K_{i,0}, N^0_{i,0}, N^1_{i,0})$, the initial state of country i, is known. The consumption good is taken to be the numéraire in each period. For any $t \geq 0$, the rental rate of capital, the price of oil, and the price of renewable energy – all in period t – are denoted, respectively, by ρ_t, ϕ_t, and φ_t. Also, the wage rate in period t in country i is denoted by $\omega_{i,t}$. The list $\left(\rho_t, (\omega_{i,t})_{i \in I}, \phi_t, \varphi_t\right)$ is called a price system in period t. A price system is a list of infinite sequences $\wp = \left(\rho_t, (\omega_{i,t})_{i \in I}, \phi_t, \varphi_t\right)_{t=0}^{\infty}$.

2.1. *The Production Technologies and Profit Maximization*

In each period t, the representative firm that produces the consumption good in country $i, i \in I$, uses the following Cobb-Douglas technology:

(1) $$Y_{i,t} = A\Omega_i(H_t)K_{i,t,1}^{\alpha}L_{i,t}^{\beta}(Q_{i,t} + B_{i,t})^{1-\alpha-\beta}.$$

In (1) $Y_{i,t}$, $K_{i,t,1}$, $L_{i,t}$, $Q_{i,t}$, and $B_{i,t}$ denote, respectively, the output net of depreciation, the capital input, the labor input, the oil input, and the renewable energy input – all in period t. As for A, it represents the technological level, while α and β are two positive parameters satisfying $\alpha + \beta < 1$. Note that $\Omega_i(H_t)$ is a scaling factor representing the impact of climate change on the production of the consumption good in country i, and this is the only factor that differentiates countries. We shall assume that the scaling factor for country i has the following functional form:

$$\Omega_i(H_t) = \begin{cases} 1 \text{ if } \underline{H} \le H_t \le H^{\#}, \\ e^{-\gamma_i(H_t - H^{\#})^2} \text{ if } H^{\#} < H_t, \end{cases}$$

where $\gamma_i > 0$ is a parameter specific to country i; H_t is the stock of greenhouse gases in the atmosphere in period t; and $\underline{H} > 0$ is the natural level of the stock of greenhouse gases in the atmosphere in steady state if there were a negligible amount of greenhouse emissions. Also, $H^{\#}$ is a constant greater than $\underline{H}$. As specified, the scaling factor for each country is 0 if the stock of greenhouse gases in the atmosphere is above its natural level, but not exceeding the critical level $H^{\#}$. Only when the stock of greenhouse gases in the atmosphere exceeds the critical level $H^{\#}$ will the negative impact of climate change begins to exert its influence. The specification is adopted so that impact of a small rise in the stock of greenhouse gases above its natural level has a no impact on the technology used in the production of the consumption good, and this means that when the stock of greenhouse gases in the atmosphere is below $H^{\#}$, the benefits obtained by burning a small amount of fossil fuels will not induce any nefarious effects due to a small change in the climate. The parameter γ_i in the scaling factor characterizes the negative impact of climate change on the consumption good technology of country i, and can be interpreted as the type of this country as far as the impact of climate change is concerned. Note that if $\gamma_i > \gamma_j$, then $\Omega_i(H_t) \le \Omega_j(H_t)$, with strict inequality holding if H_t is above the threshold level $H^{\#}$.

Given a price system $\wp = \left(r_t, (\omega_{i,t})_{i\in I}, \phi_t, \varphi_t\right)_{t=0}^{\infty}$, the representative firm that produces the consumption good in country i solves the following profit maximization in each period:

(2) $$\max_{(K_{i,t,1}, L_{i,t}, Q_{i,t}, B_{i,t}))} \begin{bmatrix} A\Omega_i(H_t)K_{i,t,1}^{\alpha}L_{i,t}^{\beta}(Q_{i,t} + B_{i,t})^{1-\alpha-\beta} \\ - r_t K_{i,t,1} - \omega_{i,t}L_{i,t} - \phi_t Q_{i,t} - \varphi_t B_{i,t} \end{bmatrix}.$$

The following first-order conditions characterize the solution of (2).

(3) $$\alpha A\Omega_i(H_t)K_{i,t,1}^{\alpha-1}L_{i,t}^{\beta}(Q_{i,t} + B_{i,t})^{1-\alpha-\beta} - \rho_t = 0,$$

(4) $\beta A\Omega_i(H_t)K_{i,t,1}^{\alpha}L_{i,t}^{\beta-1}(Q_{i,t}+B_{i,t})^{1-\alpha-\beta}-\omega_{i,t}=0,$

(5a) $(1-\alpha-\beta)A\Omega_i(H_t)K_{i,t,1}^{\alpha}L_{i,t}^{\beta}Q_{t,t}^{-\alpha-\beta}-\phi_t=0, \text{if } \phi_t<\varphi_t,$

(5b) $(1-\alpha-\beta)A\Omega_i(H_t)K_{i,t,1}^{\alpha}L_{i,t}^{\beta}B_{t,t}^{-\alpha-\beta}-\varphi_t=0, \text{if } \phi_t>\varphi_t.$

When $\phi_t=\varphi_t$, the mix of energy input $Q_{i,t}+B_{i,t}$ is indeterminate; their sum, however, is determinate, and satisfies the following relation:

(5c) $(1-\alpha-\beta)A\Omega_i(H_t)K_{i,t,1}^{\alpha}L_{i,t}^{\beta}(Q_{i,t}+B_{i,t})^{-\alpha-\beta}=\phi_t=\varphi_t.$

In any period, the energy input – measured in Btus – used by the competitive firms that produce the consumption good come either from a stock of fossil fuels or a backstop. While oil can be extracted at negligible cost, its ultimate stock is limited. The backstop, on the other hand, can produce a perpetual flow of energy. However, harnessing energy from the Sun require capital, say solar collectors. We shall assume that renewable energy is produced in the backstop sector in each country from capital according to a linear technology, with one unit of capital producing one Btu. In each period, the representative firm in the backstop sector of country i solves the following profit maximization:

(6) $\max_{K_{i,t,0}}(\varphi_t-r_t)K_{i,t,0}.$

If $r_t<\varphi_t$, then $K_{i,t,0}=\infty$. If $r_t>\varphi_t$, then $K_{i,t,0}=0$. When $r_t=\varphi_t$, $K_{i,t,0}$ is indeterminate, and must adjust to meet demand for renewable energy by the consumption good sector. Because renewable energy is produced by a linear technology that uses only capital, the capital input used to produced energy is only well determined at the global level, not at the country level. Without loss of generality, we can suppose that each country produces its own renewable energy either from its own capital or imported capital.

2.2. *The Evolution of the Stock of Greenhouse Gases*

The evolution of the stock of greenhouse gases is assumed to be governed by the following differential equation:

(7) $H_{t+1}=H_t+\sum_{i\in I}Q_{i,t}-\varepsilon(H_t-\underline{H}),$

where, we recall, $\underline{H}>0$ is the natural level of the stock of greenhouse gases in the atmosphere in steady state if there were negligible amount of greenhouse emissions, and $\varepsilon>0$ is a parameter representing the rate of decay of the excess stock of greenhouse gases above its natural level. The initial stock of greenhouse gases in the atmosphere H_0 is assumed to be known.

2.3. *Preferences and Utility Maximization*

An old individual of period t in country i owns $x_{i,t} = X_{i,t} / N^1_{i,t}$ units of oil and $k_{i,t} = K_{i,t} / N^1_{i,t}$ units of capital. Her old-age income is thus given by $\phi_t x_{,t} + (1+\rho_t)k_{i,t}$. We shall assume that she makes no bequest. Her old-age consumption is then given by $c^1_{i,t} = \phi_t x_{i,t} + (1+\rho_t)k_{i,t}$.

For a young individual of period t, a lifetime plan is a list $\left(c^0_{i,t}, c^1_{i,t+1}, x_{i,t+1}, k_{i,t+1}\right)$, where $c^0_{i,t}, c^1_{i,t+1}, x_{i,t+1}$, and $k_{i,t+1}$ represent, respectively, her current consumption, her old-age consumption, her oil investment, and her capital investment. We shall suppose that her single-period utility function of consumption is logarithmic and that she uses $\delta, 0 < \delta < 1$, as the factor to discount future utilities. Her problem of maximizing lifetime utility can be stated formally as follows:

(8) $\max_{\left(c^0_{i,t}, c^1_{i,t+1}, x_{i,t+1}, k_{i,t+1}\right)} Logc^0_{i,t} + \delta Logc^1_{i,t+1}$

subject to the following two budget constraints:

(9) $c^0_{i,t} + \phi_t x_{i,t+1} + k_{i,t+1} = \omega_{i,t}$,

(10) $c^1_{i,t+1} = \phi_{t+1} x_{i,t+1} + (1+\rho_{t+1})k_{i,t+1}$.

Now if we let

(11) $r_{t+1} = \max\left\{\dfrac{\phi_{t+1}}{\phi_t}, 1+\rho_{t+1}\right\}$,

then r_{t+1} represents the rate of return that a young individual of period t in any country earns on her savings. Using r_{t+1}, we can restate the preceding lifetime utility maximization problem in the following simpler form:

(12) $\max_{c^0_{i,t}} Logc^0_{i,t} + \delta Log[(1+r_{t+1})(\omega_{i,t} - c^0_{i,t})]$.

The solution of the preceding lifetime utility maximization problem is given by

(13) $c^0_{i,t} = \dfrac{1}{1+\delta}\omega_{i,t}$,

and the saving of a young individual of period t in country i is

(14) $s_{i,t} = \dfrac{\delta}{1+\delta}\omega_{i,t}$.

How the saving represented by (14) is divided between oil and capital investments depends on the rates of return earned by these two assets. If $\phi_{t+1}/\phi_t > 1+\rho_{t+1}$, then the individual will put all her savings in oil; that is, $x_{i,t+1} = s_{i,t}/\phi_t$ and $k_{i,t+1} = 0$. If $\phi_{t+1}/\phi_t < 1+\rho_{t+1}$, then the individual will put all her savings in capital; that is, $x_{i,t+1} = 0$ and $k_{i,t+1} = s_{i,t}$. If $\phi_{t+1}/\phi_t = 1+\rho_{t+1}$, then the individual will be indifferent between oil and capital investments. In this case, $k_{i,t+1}$ can assume any value between 0 and $s_{i,t}$.

2.4. *Definition of Competitive Equilibrium*

Let $\mathsf{P} = \left(\rho_t, (\omega_{i,t})_{i\in I}, \phi_t, \varphi_t\right)_{t=0}^{\infty}$ be *a price system. An allocation induced by* the price system P is a list of infinite sequences, say

$$\mathsf{A} = \left(c_{i,0}^1, (c_{i,t}^0, c_{i,t+1}^1, x_{i,t+1}, k_{i,t+1})_{t=0}^{\infty}, (K_{i,t,0})_{t=0}^{\infty}, (K_{i,t,1}, L_{i,t}, Q_{i,t}, B_{i,t}, Y_{i,t})_{t=0}^{\infty}, (X_{i,t}, K_{i,t}, N_{i,t}^0, N_{i,t}^1)_{t=0}^{\infty}\right)_{i\in I},$$

with the following properties: Under the price system P,

(i) $c_{i,0}^1 = [\phi_0 X_{i,0} + (1+\rho_0)K_{i,0}]/N_{i,0}^1$;

(ii) $(c_{i,t}^0, c_{i,t+1}^1, x_{i,t+1}, k_{i,t+1})$ is the optimal lifetime plan for a young individual of period t in country i;

(iii) $K_{i,t,0}$ is the demand for capital by the representative firm in the backstop sector in country i in period t;

(iv) $(K_{i,t,1}, L_{i,t}, Q_{i,t}, B_{i,t}, Y_{i,t})$ is an optimal production plan of the representative firm that produces the consumption good in country i in period t;

(v) $(X_{i,t}, K_{i,t}, N_{i,t}^0, N_{i,t}^1) = N_{t-1}^0(x_{i,t}, k_{i,t}, (1+n)N_{i,t-1}^0, 1)$.

The pair (P, A) is said to constitute *a competitive equilibrium* if the following market-clearing conditions are satisfied for each $t = 0,1,\ldots,$

(vi) $\sum_{i\in I} X_{i,t+1} + \sum_{i\in I} Q_{i,t} = \sum_{i\in I} X_{i,t}$,

(vii) $\sum_{i\in I} B_{i,t} = \sum_{i\in I} K_{i,t,0}$,

(viii) $\sum_{i\in I} [K_{i,t,0} + K_{i,t,1}] = \sum_{i\in I} K_{i,t}$,

(ix) $L_{i,t} = N_{i,t}^0$.

Observe that (vi), (vii), and (viii) represent, respectively, the equilibrium conditions on the world oil market, the world market for renewable energy, and the world market for capital. As for (ix), it represents the equilibrium conditions for the labor market in country *i*.

3. THE EXISTENCE OF COMPETITIVE EQUILIBRIUM

The existence of competitive equilibrium for an overlapping-generations model, such as the one just formulated is a delicate question, There is a sparse literature on the existence of competitive equilibrium for an overlapping-generations model of an exchange economy associated with Balasko and Shell (1980, 1981a, 1981b), Balasko, Cass, and Shell (1980), and Wilson (1981). Compared to the model formulated by these researchers, the model formulated in Section 2 is much more complex. It has a production structure, capital, and an exhaustible resource. Furthermore, the climate change that is induced by the burning of fossil fuels also generate one-way temporal production externalities – in the direction of the arrow of time – between one period and the periods that follow. Hence it is not possible to invoke the results discovered by these researchers to assert that the model formulated in Section 2 has a competitive equilibrium. The following proposition – due to Quyen (2007) – asserts the existence of a competitive equilibrium for the model formulated in Section 2; it is reproduced in the appendix of Tang (2007).

PROPOSITION 1: *There exists a competitive equilibrium under which the world's stock of fossil fuels is depleted in finite time. Furthermore, depending on the values of the parameters, there might also exist a competitive equilibrium under which part of the world's stock of fossil fuels is left in situ forever unexploited.*

4. THE TEMPORARY EQUILIBRIA: INTEREST RATES AND WAGES

In an arbitrary period $t \geq 0$, one of the following three possibilities occurs: (i) no oil is used as part of the energy input used to produce the consumption good in any country; (ii) both oil and renewable energy are used in one country to produce the consumption good; and (iii) oil is the only source of energy used to produce the consumption good in each country.

4.1. *The Energy Inputs Consist Solely of Renewable Energy*

If no oil is used in period t in country i to produce the consumption good, then renewable energy constitutes the only source of energy used in the production of the consumption good. In equilibrium, the first-order conditions (3) and (5b) now assume the following forms, respectively,

$$(15) \quad \alpha A\Omega_i(H_t)K_{i,t,1}^{\alpha-1}[N_{i,t}^0]^\beta B_{i,t}^{1-\alpha-\beta} = \rho_t,$$

and

$$(16) \quad (1-\alpha-\beta)A\Omega_i(H_t)K_{i,t,1}^{\alpha}[N_{i,t}^0]^\beta B_{t,t}^{-\alpha-\beta} = \varphi_t.$$

Furthermore, the zero profit condition in the backstop sector implies that the price of renewable energy is equal to the rental rate of capital, i.e., $\varphi_t = \rho_t$, and this result allows us to rewrite (16) as

$$(17) \quad (1-\alpha-\beta)A\Omega_i(H_t)K_{i,t,1}^{\alpha}[N_{i,t}^0]^\beta B_{t,t}^{-\alpha-\beta} = \rho_t.$$

Dividing (17) by (15), then rearranging the result, we obtain

$$(18) \quad \frac{1-\alpha-\beta}{\alpha}K_{i,t,1} = B_{i,t}.$$

Summing (18) over $i \in I$, we obtain

$$(19) \quad \begin{aligned} \frac{1-\alpha-\beta}{\alpha}\sum\nolimits_{i\in I}K_{i,t,1} &= \sum\nolimits_{i\in I}B_{i,t} \\ &= \sum\nolimits_{i\in I}K_{i,t} - \sum\nolimits_{i\in I}K_{i,t,1}. \end{aligned}$$

It follows immediately from (19) that the part of the world's capital stock used in the production of the consumption good is given by

$$(20) \quad \sum\nolimits_{i\in I}K_{i,t,1} = \frac{\alpha}{1-\beta}\sum\nolimits_{i\in I}K_{i,t}.$$

Now using (18), we can rewrite (15) as follows

$$\alpha A\left(\frac{1-\alpha-\beta}{\alpha}\right)^{1-\alpha-\beta}\Omega_i(H_t)K_{i,t,1}^{-\beta}[N_{i,t}^0]^\beta = \rho_t,$$

from which we obtain

(21) $$\rho_t^{\frac{1}{\beta}} K_{i,t,1} = \left[\alpha A\left(\frac{1-\alpha-\beta}{\alpha}\right)^{1-\alpha-\beta}\right]^{\frac{1}{\beta}} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 .$$

Summing (21) over $i \in I$, and using (20), we obtain

(22) $$\begin{aligned} \rho_t^{\frac{1}{\beta}} \sum_{i\in I} K_{i,t,1} &= \rho_t^{\frac{1}{\beta}} \frac{\alpha}{1-\beta} \sum_{i\in I} K_{i,t} \\ &= \left[\alpha A\left(\frac{1-\alpha-\beta}{\alpha}\right)^{1-\alpha-\beta}\right]^{\frac{1}{\beta}} \sum_{i\in I}\left[[\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0\right]. \end{aligned}$$

It follows from the second equality in (22) that

(23) $$\begin{aligned} \rho_t &= \alpha A\left(\frac{1-\alpha-\beta}{\alpha}\right)^{1-\alpha-\beta}\left(\frac{1-\beta}{\alpha}\right)^{\beta}\left[\frac{\sum_{i\in I} K_{i,t}}{\sum_{i\in I}[\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0}\right]^{-\beta} \\ &= \sigma_1\left(\sum_{i\in I}\eta_{i,t}\kappa_{i,t}\right)^{-\beta}, \end{aligned}$$

where we have let

(24) $$\sigma_1 = \alpha A\left(\frac{1-\alpha-\beta}{\alpha}\right)^{1-\alpha-\beta}\left(\frac{1-\beta}{\alpha}\right)^{\beta},$$

(25) $$\eta_{i,t} = \frac{N_{i,0}^0}{\sum_{j\in I}[\Omega_j(H_t)]^{\frac{1}{\beta}} N_{j,0}^0},$$

and $\kappa_{i,t} = K_{i,t} / N_{i,t}^0$, as well as have used $N_{i,t}^0 = (1+n)^t N_{i,0}^0, i \in I, t = 0,1...$

Equation (23) expresses the equilibrium interest rate in a period when no oil is used in any country to produce the consumption good as a function of the weighted average of the capital endowments per young individual of the countries that make up the world economy, with the weight of a country depending on its initial young population and the impact of climate change on its economy at the time in question.

Using (23) in (21), we obtain the following expression for the equilibrium capital input in the consumption good sector at time t in country i:

$$K_{i,t,1} = \frac{\alpha}{1-\beta}[\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 \left[\frac{\sum_{i\in I} K_{i,t}}{\sum_{i\in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0} \right]$$

(26)

$$= \frac{\alpha}{1-\beta}[\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 \sum_{j\in I} \eta_{j,t}\kappa_{j,t}.$$

Using (26) in (18), we obtain the following expression for the demand of renewable energy at time t in country i:

$$B_{i,t} = \frac{1-\alpha-\beta}{1-\beta}[\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 \left[\frac{\sum_{i\in I} K_{i,t}}{\sum_{i\in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0} \right]$$

(27)

$$= \frac{1-\alpha-\beta}{1-\beta}[\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 \sum_{j\in I} \eta_{j,t}\kappa_{j,t}.$$

The demand for capital at time t by the backstop sector in country i is then given by

$$K_{i,t,0} = \frac{1-\alpha-\beta}{1-\beta}[\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 \left[\frac{\sum_{i\in I} K_{i,t}}{\sum_{i\in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0} \right]$$

(28)

$$= \frac{1-\alpha-\beta}{1-\beta}[\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 \sum_{j\in I} \eta_{j,t}\kappa_{j,t}.$$

Using (26), (27), and (4), we obtain the following expression for the equilibrium wage rate in period t in country i:

$$\omega_{i,t} = \sigma_2[\Omega_i(H_t)]^{\frac{1}{\beta}} \left[\frac{\sum_{i\in I} K_{i,t}}{\sum_{i\in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0} \right]^{1-\beta}$$

(29)

$$= \sigma_2[\Omega_i(H_t)]^{\frac{1}{\beta}} [\sum_{j\in I} \eta_{j,t}\kappa_{j,t}]^{1-\beta},$$

where we have let

(30) $$\sigma_2 = \beta A \left[\frac{\alpha}{1-\beta}\right]^{\alpha} \left[\frac{1-\alpha-\beta}{1-\beta}\right]^{1-\alpha-\beta}.$$

Equation (29) expresses the equilibrium wage rate in a country in a period in which oil does not constitute part of the energy input used in the production of the consumption good.

4.2. *The Energy Inputs Consist Solely of Fossil Fuels*

In any period $t \geq 0$, if oil is the only source of energy used in the production of the consumption good, then $\phi_t \leq \rho_t$. The following first-order conditions characterize, respectively, the equilibrium demand for capital and the equilibrium demand for oil by the representative firm that produces the consumption good in country i:

(31) $$\alpha A \Omega_i(H_t) K_{i,t,1}^{\alpha-1} [N_{i,t}^0]^{\beta} Q_{t,t}^{1-\alpha-\beta} = \rho_t,$$

and

(32) $$(1-\alpha-\beta) A \Omega_i(H_t) K_{i,t,1}^{\alpha} [N_{i,t}^0]^{\beta} Q_{t,t}^{-\alpha-\beta} = \phi_t.$$

It follows from (31) and (32) that the equilibrium demand for oil and the equilibrium demand for capital in period t by the representative firm in the consumption good sector in country i are linked by the following relation:

(33) $$Q_{i,t} = \frac{\rho_t(1-\alpha-\beta)}{\alpha \phi_t} K_{i,t,1}.$$

Using (33) in (31), we obtain

(34) $$K_{i,t,1} = \frac{(\alpha A)^{\frac{1}{\beta}} \Omega_i(H_t) N_{i,t}^0 \left(\frac{1-\alpha-\beta}{\alpha}\right)^{\frac{1-\alpha-\beta}{\beta}}}{\phi_t^{\frac{1-\alpha-\beta}{\beta}} \rho_t^{\frac{\alpha+\beta}{\beta}}} = \sigma_3 \frac{\Omega_i(H_t) N_{i,t}^0}{\phi_t^{\frac{1-\alpha-\beta}{\beta}} \rho_t^{\frac{\alpha+\beta}{\beta}}},$$

where we have let

(35) $$\sigma_3 = (\alpha A)^{\frac{1}{\beta}} \left(\frac{1-\alpha-\beta}{\alpha}\right)^{\frac{1-\alpha-\beta}{\beta}}.$$

Using (34) in (33), we obtain the following expression for the demand for oil at each instant by the representative firm producing the consumption good in country i:

(36) $$Q_{i,t} = \sigma_4 \frac{\Omega_i(H_t) N_{i,t}^0}{\rho_t^{\frac{\alpha}{\beta}} \phi_t^{\frac{1-\alpha}{\beta}}},$$

where we have let

$$(37)\qquad \sigma_4 = (\alpha A)^{\frac{1}{\beta}} \left(\frac{1-\alpha-\beta}{\alpha} \right)^{\frac{1-\alpha}{\beta}}.$$

In period t, the equilibrium condition on the world capital market is given by

$$(38)\qquad \begin{aligned} \sum\nolimits_{i\in I} K_{i,t} &= \sum\nolimits_{i\in I} K_{i,t,1} \\ &= \frac{\sigma_3}{\phi_t^{\frac{1-\alpha-\beta}{\beta}} \rho_t^{\frac{\alpha+\beta}{\beta}}} \sum\nolimits_{i\in I} \Omega_i(H_t) N_{i,t}^0, \end{aligned}$$

where the second equality in (38) has been obtained by summing (34) over $i \in I$. Equation (38) can be rewritten as follows:

$$(39)\qquad \begin{aligned} \rho_t &= \sigma_3^{\frac{\beta}{\alpha+\beta}} \frac{1}{\phi_t^{\frac{1-\alpha-\beta}{\alpha+\beta}}} \left[\frac{\sum\nolimits_{i\in I} K_{i,t}}{\sum\nolimits_{i\in I} \Omega_i(H_t) N_{i,t}^0} \right]^{-\frac{\beta}{\alpha+\beta}} \\ &= \sigma_3^{\frac{\beta}{\alpha+\beta}} \frac{1}{\phi_t^{\frac{1-\alpha-\beta}{\alpha+\beta}}} \left[\sum\nolimits_{i\in I} \eta_{i,t} \kappa_{i,t} \right]^{-\frac{\beta}{\alpha+\beta}}. \end{aligned}$$

Using (39) in (36), we obtain

$$(40)\qquad \begin{aligned} Q_{i,t} &= \sigma_4 \frac{\Omega_i(H_t) N_{i,t}^0}{\rho_t^{\frac{\alpha}{\beta}} \phi_t^{\frac{1-\alpha}{\beta}}} \\ &= \sigma_4 \sigma_3^{\frac{\alpha}{\alpha+\beta}} \frac{\Omega_i(H_t) N_{i,t}^0}{\phi_t^{\frac{1}{\alpha+\beta}}} \left[\frac{\sum\nolimits_{i\in I} K_{i,t}}{\sum\nolimits_{i\in I} \Omega_i(H_t) N_{i,t}^0} \right]^{\frac{\alpha}{\alpha+\beta}}. \end{aligned}$$

Summing (40) over $i \in I$, we obtain the following expression of the world demand for oil in period t

$$(41)\qquad \sum\nolimits_{i\in I} Q_{i,t} = \sigma_4 \sigma_3^{\frac{\alpha}{\alpha+\beta}} \frac{\sum\nolimits_{i\in I} \Omega_i(H_t) N_{i,t}^0}{\phi_t^{\frac{1}{\alpha+\beta}}} \left[\frac{\sum\nolimits_{i\in I} K_{i,t}}{\sum\nolimits_{i\in I} \Omega_i(H_t) N_{i,t}^0} \right]^{\frac{\alpha}{\alpha+\beta}}.$$

Using (34) and (36) in the first-order condition (4), we obtain the following expression for the equilibrium wage rate in period t in country i when only oil is used in the production of the consumption good

$$\omega_{i,t} = \beta A \Omega_i(H_t)\left[\sigma_3 \frac{\Omega_i(H_t)N_{i,t}^0}{\phi_t^{\frac{1-\alpha-\beta}{\beta}} \rho_t^{\frac{\alpha+\beta}{\beta}}}\right]^{1-\beta} [N_{i,t}^0]^{\beta-1}\left[\frac{\rho_t(1-\alpha-\beta)}{\alpha\phi_t}\right]^{1-\alpha-\beta}$$

(42)

$$= \beta A\left[\frac{1-\alpha-\beta}{\alpha}\right]^{1-\alpha-\beta} \sigma_3^{1-\beta}[\Omega_i(H_t)]^{2-\beta} \frac{1}{\phi_t^{\frac{1-\alpha-\beta}{\beta}} \rho_t^{\frac{\alpha}{\beta}}}.$$

4.3. *The Energy Inputs Consist of both Oil and Renewable Energy*

In any period $t \geq 0,$ if both fossil fuels and the backstop provide energy inputs into the production of the consumption good, then $\rho_t = \phi_t = \varphi_t$. The first-order conditions (3) and (5c) then become, respectively,

(43) $$\alpha A \Omega_i(H_t) K_{i,t,1}^{\alpha-1}[N_{i,t}^0]^{\beta}(Q_{i,t} + B_{i,t})^{1-\alpha-\beta} = \rho_t,$$

and

(44) $$(1-\alpha-\beta) A \Omega_i(H_t) K_{i,t,1}^{\alpha}[N_{i,t}^0]^{\beta}(Q_{i,t} + B_{i,t})^{-\alpha-\beta} = \varphi_t = \phi_t = \rho_t.$$

Because $\phi_t = \varphi_t$, the mix of energy input $Q_{i,t} + B_{i,t}$ is indeterminate; their sum, however, is determinate, and satisfies the following relation:

(45) $$\frac{1-\alpha-\beta}{\alpha} K_{i,t,1} = Q_{i,t} + K_{i,t,0}.$$

Summing (45) over $i \in I$, we obtain

$$\frac{1-\alpha-\beta}{\alpha} \sum_{i \in I} K_{i,t,1} = \sum_{i \in I} Q_{i,t} + \sum_{i \in I} K_{i,t} - \sum_{i \in I} K_{i,t,1},$$

which can be rewritten as

(46) $$\sum_{i \in I} K_{i,t,1} = \frac{\alpha}{1-\beta}\left[\sum_{i \in I} Q_{i,t} + \sum_{i \in I} K_{i,t}\right]$$

Using (45) in (43), we obtain

$$\alpha A\left(\frac{1-\alpha-\beta}{\alpha}\right)^{1-\alpha-\beta} \Omega_i(H_t) K_{i,0,1}^{-\beta}[N_{i,0}^0]^{\beta} = \rho_t,$$

from which we obtain

$$(47)\qquad \rho_t^{\frac{1}{\beta}} K_{i,t,1} = \left[\alpha A \left(\frac{1-\alpha-\beta}{\alpha}\right)^{1-\alpha-\beta}\right]^{\frac{1}{\beta}} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 .$$

Summing (47) over $i \in I$, and using (46), we obtain

$$\rho_t^{\frac{1}{\beta}} \sum_{i\in I} K_{i,t,1} = \rho_t^{\frac{1}{\beta}} \frac{\alpha}{1-\beta}\left[\sum_{i\in I} Q_{i,t} + \sum_{i\in I} K_{i,t}\right]$$
$$= \left[\alpha A \left(\frac{1-\alpha-\beta}{\alpha}\right)^{1-\alpha-\beta}\right]^{\frac{1}{\beta}} \sum_{i\in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 ,$$

which can be rewritten as

$$(48)\qquad \rho_t^{\frac{1}{\beta}} = \left[\alpha A \left(\frac{1-\alpha-\beta}{\alpha}\right)^{1-\alpha-\beta}\right]^{\frac{1}{\beta}} \frac{\sum_{i\in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0}{\frac{\alpha}{1-\beta}\left[\sum_{i\in I} Q_{i,t} + \sum_{i\in I} K_{i,t}\right]} .$$

Using (48) in (47), we obtain the following expression for the demand for capital in the consumption good sector of country i in period t:

$$(49)\qquad K_{i,t,1} = \frac{\alpha}{1-\beta} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 \frac{\sum_{i\in I} Q_{i,t} + \sum_{i\in I} K_{i,t}}{\sum_{i\in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0} .$$

Using (49) and (45) in the first-order condition (4), we obtain the following expression for the wage rate in period t in country i when both fossil fuels and renewable energy are used in the production of the consumption good:

$$(50)\qquad \omega_{i,t} = \beta A \left[\frac{1-\alpha-\beta}{\alpha}\right]^{1-\alpha-\beta} \left[\frac{\alpha}{1-\beta}\right]^{1-\beta} [\Omega_i(H_t)]^{\frac{1}{\beta}} \left[\frac{\sum_{i\in I} Q_{i,t} + \sum_{i\in I} K_{i,t}}{\sum_{i\in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0}\right]^{1-\beta} .$$

5. A CHARACTERIZATION OF THE COMPETITIVE EQUILIBRIA

To study the climate change induced by the burning of fossil fuels, we shall assume that $\sum_{i\in I} X_{i,0}$, the world's initial stock of fossil fuels, is much greater than $\sum_{i\in I} K_{i,0}$, the world's initial capital stock.[3] Intuitively, we expect that if the world's initial stock of fossil fuels is large, but the world's initial capital stock is not, then the oil input into the production of the consumption good will be large, and the backstop sector in each country is not active. The following lemma, which is a generalization to a multi-country world of (i) of Lemma 3 found in Hung and Quyen (2007), confirms this intuition.

LEMMA 1: *If the world's initial stock of fossil fuels is large, then the initial global demand for oil will be high, and the backstop sector in each country will not be active in period 0.*

PROOF: There are three possibilities to consider: (i) no oil is used as part of the energy input used to produce the consumption good in any country in period 0, (ii) both oil and renewable energy are used in one country to produce the consumption good in period 0, and (iii) oil is the only source of energy used to produce the consumption good in period 0.

If no oil is used in any country to produce the consumption good in period 0, then we must have $\phi_0 \geq \rho_0$. Furthermore, applying (23) for $t = 0$, we can assert that

$$\text{(51)} \qquad \rho_0 = \sigma_1 (\textstyle\sum_{i \in I} \eta_{i,0} \kappa_{i,0})^{-\beta} \leq \phi_0,$$

i.e., the price of oil is bounded below by $\sigma_1 (\sum_{i \in I} \eta_{i,0} \kappa_{i,0})^{-\beta}$, On the other hand, according to (29), the wage rate in period 0 in country i is given by

$$\text{(52)} \qquad \omega_{i,0} = \sigma_2 [\Omega_i (H_0)]^{\frac{1}{\beta}} [\textstyle\sum_{j \in I} \eta_{j,0} \kappa_{j,0}]^{1-\beta},$$

which is bounded above. Thus id the world's initial stock of fossil fuels is large and is not exploited in period 0, the value of this stock will exceed the labor income of all the young individuals of period 0 in the whole world. Hence if the world's initial stock of fossil fuels is large, it will be exploited in period 0.

Under possibility (ii), we have $\phi_0 = \rho_0$, and applying (49) for $t = 0$, we can write

$$\text{(53)} \qquad K_{i,0,1} = \frac{\alpha}{1-\beta} [\Omega_i (H_0)]^{\frac{1}{\beta}} N_{i,0}^0 \frac{\sum_{i \in I} Q_{i,0} + \sum_{i \in I} K_{i,0}}{\sum_{i \in I} [\Omega_i (H_0)]^{\frac{1}{\beta}} N_{i,0}^0} < \textstyle\sum_{i \in I} K_{i,0}.$$

The strict inequality in (53) implies that if possibility (ii) holds, then $\sum_{i \in I} Q_{i,0}$ is bounded above when the world's initial stock of fossil fuels becomes indefinitely large. Applying (48) and (50), respectively, for $t = 0$, we obtain

[3] The burning of a small stock of fossil fuels has a negligible impact on the climate.

(54) $$\rho_0^{\frac{1}{\beta}} = \left[\alpha A \left(\frac{1-\alpha-\beta}{\alpha}\right)^{1-\alpha-\beta}\right]^{\frac{1}{\beta}} \frac{\sum_{i\in I}[\Omega_i(H_0)]^{\frac{1}{\beta}} N_{i,0}^0}{\frac{\alpha}{1-\beta}\left[\sum_{i\in I} Q_{i,0} + \sum_{i\in I} K_{i,0}\right]}.$$

and

(55) $$\omega_{i,0} = \beta A \left[\frac{1-\alpha-\beta}{\alpha}\right]^{1-\alpha-\beta} \left[\frac{\alpha}{1-\beta}\right]^{1-\beta} [\Omega_i(H_0)]^{\frac{1}{\beta}} \left[\frac{\sum_{i\in I} Q_{i,0} + \sum_{i\in I} K_{i,0}}{\sum_{i\in I}[\Omega_i(H_0)]^{\frac{1}{\beta}} N_{i,0}^0}\right]^{1-\beta}.$$

Note that if possibility (ii) holds, then (54) is bounded below, and (55) bounded above, which together imply that the wages earned by all the young individuals of period 0 in the world cannot afford to buy the world's remaining stock of fossil fuels $[\sum_{i\in I} X_{i,0}] - [\sum_{i\in I} Q_{i,0}]$ for investment purposes. Hence possibility (ii) cannot arise in equilibrium.

Having proved that possibility (iii) will prevail if the world's initial stock of fossil fuels is large, let us now show that the amount of oil extracted for use in the production of the consumption good in period 0 will be large. Indeed, if there exists a number $M > 0$ such that $\sum_{i\in I} Q_{i,0} < M$ no matter how large $\sum_{i\in I} X_{i,0}$ is, then we must have

(56) $$\begin{aligned} \phi_0\left([\textstyle\sum_{i\in I} X_{i,0}] - M\right) &< \phi_0\left([\textstyle\sum_{i\in I} X_{i,0}] - [\textstyle\sum_{i\in I} Q_{i,0}]\right) < \textstyle\sum_{i\in I} Y_{i,0} \\ &< \textstyle\sum_{i\in I} A\Omega_i(H_0)\left[\sum_{i\in I} K_{i,0}\right]^{\alpha} [N_{i,t}^0]^{\beta} \left[M + \sum_{i\in I} K_{i,0}\right]^{1-\alpha-\beta}. \end{aligned}$$

In (56), the strict inequality between the expression on the left-hand side of the first inequality and the expression on the last line implies that ϕ_0 will be small when $\sum_{i\in I} X_{i,0}$ is large. Applying (40) for $t = 0$, we can then assert that $Q_{i,0}$ will be large when $\sum_{i\in I} X_{i,0}$ is large, contradicting the premise that $\sum_{i\in I} Q_{i,0}$ is bounded above by M. ■

According to Lemma 1, if the world's initial stock of fossil fuels is large, then there will be excessive burning of fossil fuels under the competitive equilibrium. The excessive burning of fossil fuels leads to a dramatic rise in the stock of greenhouse gases in the atmosphere in period 1, which shifts downward the production function of the consumption good in every country in period 1, with the ensuing result of a drastic reduction in the output of the consumption good in each country in period 1. The following proposition, which is a generalization of Tang (2007, Proposition 3) to a multi-country world, confirms this result.

PROPOSITION 2: *If the initial world's stock of fossil fuels is large, then under the competitive equilibrium there will be excessive burning of fossil fuels in period 0, with the ensuing consequence of a drastic reduction in the output of the consumption good in each country in period 1. More precisely,* $Y_{i,1} \to 0$ *for each* $i \in I$ *when* $\sum_{i\in I} X_{i,0} \to +\infty$.

PROOF: The saving of an individual in country i in period 0 is

$$(57) \qquad s_{i,0} = \frac{\delta}{1+\delta}\omega_{i,0} = \frac{\delta}{1+\delta}\beta A\Omega_i(H_0)K^{\alpha}_{i,0,1}L^{\beta-1}_{i,0}(Q_{i,0}+B_{i,0})^{1-\alpha-\beta}$$

Because the saving of a young individual in country i of period 0 must be at least sufficient to purchase the oil stock remaining at the end of period 0, the following inequality must hold:

$$(58) \qquad Q_{i,0} \geq \frac{(1+\delta)(1-\alpha-\beta)X_0 - \delta\beta L^{-1}_{i,0}B_{i,0}}{\delta\beta L^{-1}_{i,0} + (1+\delta)(1-\alpha-\beta)}$$

It follows from (58) that the remaining oil stock at the beginning of period 1 satisfies the following inequality

$$(59) \qquad X_{i,1} = X_{i,0} - Q_{i,0} \leq \frac{\delta\beta L^{-1}_{i,0}}{\delta\beta L^{-1}_{i,0} + (1+\delta)(1-\alpha-\beta)}(X_{i,0}+B_{i,0})$$

The capital investment of a young individual of country i of period 0 was given by

$$(60) \qquad \begin{aligned} K_{i,1} &= s_{i,0} - \phi_0 X_{i,1} \\ &= A\Omega_i(H_0)K^{\alpha}_{i,0,1}L^{\beta}_{i,0}(Q_{i,0}+B_{i,0})^{-\alpha-\beta}\left[\begin{aligned}&\left(\frac{\delta}{1+\delta}\beta L^{-1}_{i,0}+1-\alpha-\beta\right)Q_{i,0} \\ &+\frac{\delta}{1+\delta}\beta L^{-1}_{i,0}B_{i,0} - (1-\alpha-\beta)X_{i,0}\end{aligned}\right] \end{aligned}$$

Now the output of the consumption good of country i in period 1 was

$$Y_{i,1} = Ae^{-\gamma_i\left[(1-\varepsilon)H_0 + \sum_{i\in I} Q_{i,0}\right]^2} K^{\alpha}_{i,1,1}L^{\beta}_{i,1}(Q_{i,1}+B_{i,1})^{1-\alpha-\beta}, \quad \text{assuming } \underline{H} = 0.$$

Because $Q_{i,1} \leq X_{i,1}$ and $B_{i,1} \leq K_{i,1}$ the output of the consumption good in period 1 satisfied:

(61)

$$Y_{i,1} \le Ae^{-\gamma_i\left[(1-\varepsilon)H_0+\sum_{i\in I}Q_{i,0}\right]^2} K_{i,1,1}^{\alpha}L_{i,1}^{\beta}\left[\begin{array}{l}\dfrac{\delta\beta L_{i,0}^{-1}}{\delta\beta L_{i,0}^{-1}+(1+\delta)(1-\alpha-\beta)}(X_{i,0}+B_{i,0})+ \\ +A\Omega_i(H_0)K_{i,0,1}^{\alpha}L_{i,0}^{\beta}(Q_{i,0}+B_{i,0})^{-\alpha-\beta}\left[\begin{array}{l}\left(\dfrac{\delta}{1+\delta}\beta L_{i,0}^{-1}+1-\alpha-\beta\right)Q_{i,0}+ \\ +\dfrac{\delta}{1+\delta}\beta L_{i,0}^{-1}B_{i,0}-(1-\alpha-\beta)X_{i,0}\end{array}\right]\end{array}\right]^{1-\alpha-\beta}$$

and $Q_{i,0} \le X_{i,0}$, it followed from the preceding inequality that

$$(62) \qquad Y_{i,1} \le Ae^{-\gamma_i\left[(1-\varepsilon)H_0+\sum_{i\in I}Q_{i,0}\right]^2} \times K_{i,1,1}^{\alpha}L_{i,1}^{\beta}\left[\begin{array}{l}\dfrac{\delta\beta L_{i,0}^{-1}}{(1+\delta)(1-\alpha-\beta)}\left(Q_{i,0}+B_{i,0}\right)+ \\ +\dfrac{\delta}{1+\delta}\beta A\Omega_i(H_0)K_{i,0,1}^{\alpha}L_{i,0}^{\beta-1}(Q_{i,0}+B_{i,0})^{1-\alpha-\beta}\end{array}\right]^{1-\alpha-\beta}$$

Now when $Q_{i,0}$ was large, the exponential term $e^{-\gamma_i\left[(1-\varepsilon)H_0+\sum_{i\in I}Q_{i,0}\right]^2}$ would dominate the expression $\left[\begin{array}{l}\dfrac{\delta\beta L_{i,0}^{-1}}{(1+\delta)(1-\alpha-\beta)}\left(Q_{i,0}+B_{i,0}\right)+ \\ +\dfrac{\delta}{1+\delta}\beta A\Omega_i(H_0)K_{i,0,1}^{\alpha}L_{i,0}^{\beta-1}(Q_{i,0}+B_{i,0})^{1-\alpha-\beta}\end{array}\right]^{1-\alpha-\beta}$. Therefore, the output of the consumption good of each country in period 1 under the competitive equilibrium would tend to 0 when its initial oil stock tended to infinity. ■

Now along a competitive equilibrium path, the resource price must appreciate at the rate of interest in order to induce a young individual to put part of her savings in oil. Furthermore, whenever oil is extracted for use in the production of the consumption good, its price is constrained not to exceed the rental rate of capital. Hence extraction activities cannot be expected to go on forever. The following lemma confirms this intuition.

LEMMA 2: *Under a competitive equilibrium, the exploitation of the world's stock of fossil fuels is terminated in finite time.*

PROOF: If the exploitation of the world's stock of fossil fuels is not terminated in finite time, then there exists a strictly increasing of natural numbers, say $\tau(m), m = 0,1,...,$ such that the world's stock of fossil fuels is exploited in period $\tau(m)$ to meet the world's demand, i.e., $\sum_{i\in I} Q_{i,\tau(m)} > 0$. Because the successive young generations of periods $t = 0,1,...$ invest in both oil and capital, the price of oil must rise geometrically through time at the rate of interest. Hence $\phi_{\tau(m)} = \phi_0 \prod_{t=1}^{\tau(m)} (1+\rho_t)$, and this means $\phi_{\tau(m)}$ is strictly increasing with m. We claim that $\ell im_{m\to\infty} \phi_{\tau(m)} = \infty$. Indeed, if $\ell im_{m\to\infty} \phi_{\tau(m)} < \infty$, then $\ell im_{m\to\infty} \rho_{\tau(m)} = 0$, which implies $\rho_{\tau(m)} < \phi_{\tau(m)}$ for large m, and this last strict inequality means that oil will not constitute part of the energy input used to produce the consumption good in period $\tau(m)$, contradicting the reductio ad absurdum hypothesis. The claim is now proved. To complete the proof of Lemma 2, pick a a positive integer m such that $\phi_{\tau(m)} > 1$, then note that

$$\phi_{\tau(m+1)} = \phi_{\tau(m)} \prod_{t=\tau(m)+1}^{\tau(m+1)} (1+\rho_t) > (1+\rho_{\tau(m+1)}) > \rho_{\tau(m+1)},$$

which means that oil does not constitute part of the energy input used to produce the consumption good in period $\tau(m+1)$, contradicting the premise of the reduction ad absurdum argument. ■

In light of Lemmas 1 and 2, we can characterize a competitive equilibrium as consisting of three phases. In the first phase, fossil fuels provide all the energy needs of the world economy. During this phase, the price of oil rises steadily at the rate of interest. The second phase – which might or might not exist – begins when the price of oil has risen to the level of the interest rate. In this phase, the two technologies – fossil fuels and backstop – co-exist, and the energy inputs used in the production of the consumption good consist of both oil and renewable energy. In the third phase of the competitive equilibrium – the post fossil fuel phase – the backstop completely takes over and provides all the energy needs of the world economy. We shall now analyze these three phases in the reverse order.

5.1. *The Post Fossil Fuel Phase*

Let T denote the time period that follows immediately the period in which the world's stock of fossil fuels is last exploited. Then we have $\sum_{i\in I} Q_{i,T-1} > 0$ and $\sum_{i\in I} Q_{i,t} = 0, t \geq T$. While Lemma 2 asserts that extraction activities are terminated in finite time, it is silent about whether the world's stock of fossil fuels is depleted when extraction activities are terminated. Depending on the values of the parameters, it might be the case that $\sum_{i\in I} X_{i,T} > 0$, i.e., the world's stock of fossil fuels is only partially depleted. Thus there are two possibilities to consider: complete oil exhaustion and partial oil exhaustion.

5.1.1. *Complete Oil Exhaustion*

After the world's stock of fossil fuels has been depleted, the energy input used to produce the consumption good in each country in each period consists only of renewable energy. According to (29), the equilibrium wage rate in period $t \geq T$ in country i is given by

$$(63) \qquad \omega_{i,t} = \sigma_2[\Omega_i(H_t)]^{\frac{1}{\beta}}[\sum_{j\in I}\eta_{j,t}\kappa_{j,t}]^{1-\beta}, \qquad (i \in I, t \geq T).$$

The capital endowment of a young individual of period $t, t \geq T$, in country i is then given by

$$(64) \qquad \kappa_{i,t+1} = \frac{\delta\sigma_2}{(1+\delta)(1+n)}[\Omega_i(H_t)]^{\frac{1}{\beta}}[\sum_{j\in I}\eta_{j,t}\kappa_{j,t}]^{1-\beta}.$$

Multiplying (64) by $\eta_{i,t+1}$ then summing $\eta_{i,t+1}\kappa_{i,t+1}$ over $i, i \in I$, we obtain

$$(65) \qquad \sum_{i\in I}\eta_{i,t+1}\kappa_{i,t+1} = \frac{\delta\sigma_2}{(1+\delta)(1+n)}\left(\sum_{i\in I}\eta_{i,t+1}[\Omega_i(H_t)]^{\frac{1}{\beta}}\right)[\sum_{j\in I}\eta_{j,t}\kappa_{j,t}]^{1-\beta}.$$

.

Now note that when $t \to \infty$, we have $H_t \to \underline{H}$, which implies that when t is large enough so that $H_t < H^{\#}$, we have $\sum_{i\in I}\eta_{i,t+1}[\Omega_i(H_t)]^{\frac{1}{\beta}} = 1$, and $\sum_{i\in I}\eta_{i,t+1}\kappa_{i,t+1}$ must converge. More specifically, we have

$$(66) \qquad \ell im_{t\to\infty}\sum_{i\in I}\eta_{i,t+1}\kappa_{i,t+1} = \bar{\kappa} = \left[\frac{\delta\sigma_2}{(1+\delta)(1+n)}\right]^{\frac{1}{\beta}}.$$

Using (66) in (65), we obtain

$$(67) \qquad \ell im_{t\to\infty}\kappa_{i,t} = \bar{\kappa}, \qquad (i \in I).$$

Equation (67) asserts that in the long run – after the negative impact of climate change has dissipated – and under the scenario of complete oil exhaustion the capital endowment per young individual in each country converges to the same steady-state level, and this steady-state level depends on the production technologies, the preferences, and the population growth rate.

5.1.2. *Incomplete Oil Depletion*

Suppose that at the end of the period in which the world's stock of fossil fuels is last exploited the remaining stock of oil is positive, i.e., $\sum_{i\in I}X_{i,T} > 0$. Under this scenario, the remaining stock in situ $\sum_{i\in I}X_{i,T} > 0$ is passed on from one generation to the next for $t \geq T$. Although now the expressions for the equilibrium interest rate and the equilibrium wage rate in each country are still the same as under complete oil exhaustion, the savings of a young individual of each period $t \geq T$ must include both capital and oil, and the capital endowment per young individual in country i in the next period is given by

$$(68)\qquad \kappa_{i,t+1} = \frac{1}{1+n}\left[\frac{\delta\sigma_2}{(1+\delta)}[\Omega_i(H_t)]^{\frac{1}{\beta}}[\textstyle\sum_{j\in I}\eta_{j,t}\kappa_{j,t}]^{1-\beta} - \phi_t x_{i,t+1}\right].$$

For a period t far into the future, we have $H_t < H^{\#}$, and all the negative impact of climate change has dissipated. In such a period, we have $\Omega_i(H_t) = 1$, $\eta_{i,t} = 1$, and this means that the capital endowment per young individual, the oil endowment per young individual, and the wage rate are identical across countries. Thus when t is large, the evolution of the world economy can be studied by focusing on an arbitrary country, say country i. Furthermore, equation (68) is now reduced to

$$(69)\qquad \kappa_{i,t+1} = \frac{1}{1+n}\left[\frac{\delta\sigma_2}{(1+\delta)}\kappa_{i,t}^{1-\beta} - \phi_t x_{i,t+1}\right].$$

Let us consider an arbitrary country i and an arbitrary, but large period t. We have $x_{i,t} = [\sum_{i\in I} X_{i,T}]/\sum_{i\in I} N_{i,t-1}$, which implies that $x_{t+1} = x_t/(1+n)$. If the system converges to a steady state, let

$$(70)\qquad \hat{\kappa} = \ell im_{t\to\infty}\kappa_{i,t},$$

and

$$(71)\qquad z = \ell im_{t\to\infty}\phi_t x_{i,t+1}.$$

Because the value of the oil investment made by a young individual in steady state remains the same, and because the amount of oil in the investment portfolio of a young individual declines geometrically at the rate of population growth, the price of oil must rise geometrically in steady state at rate n, which in turn implies that the rate of interest in steady state must be equal to n. That is, $\sigma_1\hat{\kappa}^{-\beta} = n$, or equivalently

$$(72)\qquad \hat{\kappa} = \left[\frac{\sigma_1}{n}\right]^{\frac{1}{\beta}}.$$

Using (72) in the steady-state version of (69), we obtain

$$(73)\qquad z = \left[\frac{\sigma_1}{n}\right]^{\frac{1}{\beta}}\left(\frac{\delta\sigma_2}{1+\delta}\left[\frac{\sigma_1}{n}\right]^{-1} - (1+n)\right).$$

In order for z to be positive, the following condition must be satisfied:

$$(74)\qquad \frac{n}{n+1} > \frac{(1+\delta)\sigma_1}{\delta\sigma_2}.$$

Note that (74) cannot be satisfied if $[\sigma_1(1+\delta)]/[\sigma_2\delta] \geq 1$ or if the population growth rate is small.

Now recall that under complete oil exhaustion the stead-state capital labor ratio is given by

$$\bar{\kappa} = \left[\frac{\delta\sigma_2}{(1+\delta)(1+n)}\right]^{\frac{1}{\beta}} \tag{75}$$

$$= \left[\frac{\sigma_1}{n}\right]^{\frac{1}{\beta}} \left[\frac{n\delta\sigma_2}{\sigma_1(1+\delta)(1+n)}\right]^{\frac{1}{\beta}} > \left[\frac{\sigma_1}{n}\right]^{\frac{1}{\beta}} = \hat{\kappa},$$

where the strict inequality in (75) has been obtained with the help of (74). The steady-state capital labor ratio is thus lower under incomplete than under complete oil exhaustion.

The equilibrium under incomplete oil exhaustion is clearly inefficient: the part of the world's stock of fossil fuels left in situ can be used judiciously to increase the output of the consumption good without inducing any negative impact of climate change after the stock of greenhouse gases has fallen below its threshold level.

The climate change has been warned to have long-term impacts on an economy even after the world's stock of fossil fuels is last exploited. The following section investigates the economic impacts of climate change in different countries.

5.1.3. Capital Flows and Welfare

According to (19), the wage rate earned by a young individual after oil exhaustion is lower in a country which is more adversely affected (higher value of γ_i) by climate change. On the other hand, free capital mobility means that a young individual – regardless of country of origin – earns the same rate of return for her savings. Hence the lifetime utility of an individual varies inversely with the negative impact of climate change on her country of origin. This result is formally stated in the following proposition.

PROPOSITION 3: *For each generation that is born after the world's stock of fossil fuels has been depleted, the welfare of each of its members varies across countries inversely with the negative impact of climate change on its own country of origin.*

To study the impact of climate change on the direction of capital flows, let $\hat{k}_{i,t} = (K_{i,t,0} + K_{i,t,1})/N^0_{i,t}, i \in I, t \geq T$, denote the capital demand young individual ratio of country i in a period after the world's stock of fossil fuels has been depleted, then:

(76)

$$\hat{k}_{i,t+1} - \kappa_{i,t+1} = \frac{\delta\sigma_2}{(1+\delta)(1+n)} e^{-\frac{\gamma_i}{\beta}(H_{t+1}-\underline{H})^2} [\sum_{j\in I} \eta_{j,t} k_{j,t}]^{1-\beta} \left[\frac{\sum_{i\in I} N_{i,0}^0 e^{-\frac{\gamma_i}{\beta}(H_t-\underline{H})^2}}{\sum_{j\in I} e^{-\frac{\gamma_j}{\beta}(H_{t+1}-\underline{H})^2} N_{j,0}^0} - e^{-\frac{\gamma_i}{\beta}(H_t-H_{t+1})^2} \right].$$

The sign of (76) is the same as the sign of $\frac{\sum_{i\in I} N_{i,0}^0 e^{-\frac{\gamma_i}{\beta}(H_t-\underline{H})^2}}{\sum_{j\in I} e^{-\frac{\gamma_j}{\beta}(H_{t+1}-\underline{H})^2} N_{j,0}^0} - e^{-\frac{\gamma_i}{\beta}(H_t-H_{t+1})^2}$, an expression that depends on the stock of greenhouse gases in the atmosphere, the parameters that characterize the negative impact of climate change on the production of the consumption good, the initial populations of the countries that make up the world economy, and the factor share of labour in national income of each country. Note that

$$0 < \frac{\sum_{i\in I} N_{i,0}^0 e^{-\frac{\gamma_i}{\beta}(H_t-\underline{H})^2}}{\sum_{j\in I} e^{-\frac{\gamma_j}{\beta}(H_{t+1}-\underline{H})^2} N_{j,0}^0} < 1.$$

and $e^{-\frac{\gamma_i}{\beta}(H_t-H_{t+1})^2}$ is close to 0 if γ_i is large, and is close to 1 if γ_i is small. Hence $\hat{k}_{i,t} - \kappa_{i,t}$ is positive (negative) if γ_i is large (small). The following proposition is now immediate:

PROPOSITION 4: *In any period after the world's stock of fossil fuels has been depleted, a country that is severely affected by climate change will import capital, while a country that is impervious to climate change will export capital.*

5.2. *The Phase of Technology Substitution*

In order to induce a young individual to put part of her savings in oil, the price of oil must rise steadily at the rate of interest but not exceed the rental rate of capital. Moreover, due to the ultimate finite stock of fossil fuels, it is necessary to look for an everlasting source of energy input for substitution in the long run. Hence when the price of oil has risen to the level of the interest rate the backstop will be brought into use. The transition phase in which both fossil fuels and backstop technologies co-exist might arise. If fossil fuels and the backstop are both used in a period, say period $t+1$, then $\rho_{t+1} = \phi_{t+1} = \varphi_{t+1}$ and the individual will be indifferent between oil and backstop capital investments. Therefore, the following condition satisfies:

(77) $\phi_{t+1} = (1+\rho_{t+1})\phi_t = \rho_{t+1}$,

which implies that $\phi_t < 1$, that is, the price of oil cannot be too high.

5.3. *The Age of Fossil Fuels*

Along the equilibrium path the rate of return to capital must be greater than or equal to the rate of return to oil investment. If oil is the only source of energy used in the production of the consumption good in a period, say period $t+1$, then $\phi_{t+1} < \rho_{t+1}$ and the two rates of return are equal. Hence, the following condition satisfies:

(78) $\quad \phi_{t+1} = (1+\rho_{t+1})\phi_t < \rho_{t+1}$

(77) and (78) show that the price of oil cannot be too high during the first and the second phases of the competitive equilibrium and has risen steadily to and then exceed the level of the rental rate of capital when the oil stock has been depleted.

6. NUMERICAL EXAMPLES

In the numerical examples, the following values for the parameters of the model are assumed:

$$I = 2; A = 4; \alpha = 0.25; \beta = 0.65; \varepsilon = 0.05; \delta = 0.8;$$
$$\gamma_1 = 0.25; \gamma_2 = 0.75; n = 0.01; \underline{H} = 0.01; H^{\#} = 0.25$$

A numerical illustration of a competitive equilibrium consisting of three phases with complete oil exhaustion is presented in the following table:

Period	X_t	K_t	$N^0_{1,t}$	$N^0_{2,t}$	H_t	ϕ_t	ρ_t	$\omega_{1,t}$	$\omega_{2,t}$	$\kappa_{1,t}$	$\kappa_{2,t}$
0	0.879	0.014	0.98	1.225	0.01	0.376	38.7	0.64	0.64		
1	0.299	0.277	0.99	1.238	0.59	0.603	0.603	1.2	1.1		
2	0.11	1.07	1	1.25	0.75	1.518	1.518	1.64	1.35	0.5	0.455
3	0	1.48	1.01	1.263	0.82	3.445	1.269	1.75	1.36	0.721	0.595
4	0	1.548	1.02	1.275	0.78	7.811	1.267	1.78	1.44	0.77	0.598
…	…	…	…	…	…	…	…	…	…	…	…
100	0	5.314	2.65	3.314	0.01	8.37×10^{33}	1.224	2.03	2.03	0.891	0.891
101	0	5.371	2.68	3.348	0.01	1.86×10^{34}	1.224	2.03	2.03	0.891	0.891

A numerical example of a competitive equilibrium without the phase of technology substitution with complete oil exhaustion is illustrated in the following table:

Period	X_t	K_t	$N^0_{1,t}$	$N^0_{2,t}$	H_t	ϕ_t	ρ_t	$\omega_{1,t}$	$\omega_{2,t}$	$\kappa_{1,t}$	$\kappa_{2,t}$
0	0.91	1.56 $\times 10^{-6}$	0.98	1.23	0.01	0.053	34688	0.064	0.064		
1	0.5	0.07	0.99	1.24	0.42	0.64	11.16	0.95	0.93		
2	0	1.25	1	1.25	0.9	1.5	1.34	1.64	1.18	0.6	0.52
3	0	1.38	1.01	1.26	0.86	3.45	1.3	1.7	1.28	0.72	0.521
4	0	1.49	1.02	1.28	0.81	7.86	1.28	1.75	1.37	0.75	0.565
…	…	…	…	…	…	…	…	…	…	…	…
100	0	5.31	2.65	3.31	0.01	8.57 $\times 10^{33}$	1.224	2.03	2.03	0.891	0.891
101	0	5.37	2.68	3.35	0.01	1.91 $\times 10^{34}$	1.224	2.03	2.03	0.891	0.891

Note that the initial distribution of wealth (global capital stock and global oil stock) among the old generations of period 0 only affects the welfare of the old generations of period 0. A redistribution of capital and oil between the old generations of the two countries in period 0 has no impact on the equilibrium price system , and a fortiori no impact on the welfare of generations of period 1, 2, 3, …. Moreover, the price of oil is monotonically increasing while the convergence of rental rate of capital is not monotonic.

7. CONCLUSIONS

The new feature in this paper is the analysis of the economic growth for an economy in which the production technology allows for the substitution of an exhaustible, says oil, by an everlasting source of energy, says energy from the Sun, for a sustainable development. An overlapping-generations framework with the simplest specification possible was adopted. In the model, an individual agent has a two-period life caring for her young and old age consumptions, and transfers her wealth over time through either oil or capital asset, or both. This modeling strategy is thus somewhat more "market oriented" than the central planning model with perfect foresight in the Ramsey-Solow tradition. In the light of an interesting finding that the oil stock might not be entirely depleted and the unused part in situ serves the role of storing value as money, there are infinitely many equilibria depending on the data that characterize the initial state of an economy. In contrast with paper money which has no intrinsic value, oil is a factor of production. Therefore, leaving productive oil in situ as a bubble is clearly inefficient. In order to restore economic efficiency, the obvious policy implication is how to induce complete resource depletion by some public intervention which should, at some point in time, discourage asset holding under the form of in situ resource. Because of the general equilibrium repercussion of any policy on the rest of the economy, this task is not that simple and should be analyzed in future research. This paper formulates a model with no controls on the emissions of greenhouse gases. The next step is to explain why and how a self-enforcing international environmental treaty should be negotiated.

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