Central Bank Credibility Under Inferential Expectations

Gordon D. Menzies
University of Technology Sydney &
Centre for Applied Macroeconomic Analysis

Daniel J. Zizzo
University of East Anglia &
Centre for Applied Macroeconomic Analysis

Timo Henckel*
Australian National University &
Centre for Applied Macroeconomic Analysis

Nick Prokhovnik
University of Technology Sydney

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Abstract

The theory of inferential expectations states that "economic agents hold beliefs that are subject to falsification by new information, in much the same way that they are in conventional statistical hypothesis testing." We outline the role expectations play in current monetary economic theory by

*Corresponding author. Contact details: Crawford School of Economics and Government, Australian National University, Canberra ACT 0200, Australia; Tel: +61-2-61255540; Fax: +61-2-61255570; Email: timo.henckel@anu.edu.au.

incorporating inferential expectations into the Barro-Gordon model (1983) of time inconsistency. In a simple version of the model, the time consistent inflation target range shrinks as the duration of cheating increases, where time is a function of agents' test size, α. In the second version of the model both magnitude and time affect the monetary authority's incentive to cheat. The model produces two key insights: i) monetary authorities may cheat for some time without being caught due to inflation noise and ii) announcing a target range is optimal.

1. Introduction

The rational expectations (RE) hypothesis is still by far the most dominant form of expectations formation used in economics. Its widespread use notwithstanding, criticism of RE has grown and several authors have sought better ways of describing how people form expectations. Recent work by Menzies & Zizzo (2005) introduced a new type of expectations mechanism whereby, "economic agents hold beliefs that are subject to falsification by new information, in much the same way that they are in conventional statistical hypothesis testing." This is dubbed "inferential expectations." This idea has several appealing features: it allows for a formal way of describing how economic agents might change their beliefs in an uncertain world and it describes how sudden changes in expectations occur over seemingly trivial pieces of information.

We take the idea of inferential expectations and apply it to the Barro-Gordon (1983) model of time inconsistency.

The paper proceeds as follows. The second section provides a short overview of the theory of inferential expectations. The third section briefly reviews the original Barro-Gordon (1983) model, focusing on the benefits of a strong reputation in sustaining low inflation rates. The model is well-suited for including inferential expectations—it incorporates uncertainty and strategic behavior and it is simple.

The fourth section introduced IE into the Barro- Gordon model. Time becomes a test statistic to decide whether departures from an announced inflation target are temporary or permanent. Given a sampling distribution for time, the range of credible inflation announcements decreaseas as the number of periods the monetary authority can cheat for increases. The fifth augments the IE model, taking into account the magnitude and time of the cheat. The model delivers a formula that the monetary authority could use to enable it to cheat without being 'caught'. It also demonstrates that under inferential expectations an announced inflation target *range* is optimal. The sixth section discusses two further issues—the problem agents face when the population standard deviation is unknown and the relationship between inferential expectations and central bank credibility.

2. Inferential Expectations: A New Theory of Belief Formation

The rational expectations (RE) hypothesis exerts a powerful hold on the economics profession but recently theoreticians have started formulating alternatives.

Examples include (1) near rationality (Akerlof and Yellen, 1985), (2) parameter uncertainty and econometric learning (Evans and Honkapohja, 2001), (3) model uncertainty and robustness (Hansen and Sargent, 2001), (4) information processing constraints and 'rational inattention' (Sims, 2003) and (5) utility-based beliefs, or 'optimal' expectations (Brunnermeier and Parker, 2005).[1]

While economists have become uneasy with RE in recent decades, a parallel debate has been carried on in the physical sciences on the status of hypothesis testing as a model of belief formation. Mayo (1996) and Mayo and Spanos (2006) have argued that there is a paradoxical reluctance of scientists to describe their own belief changes in terms of Neyman-Pearson testing. A Bayesian account is more popular, despite the widespread use of hypothesis testing by scientific practitioners (Chang 1997). The crux of the argument is that the actual practice of scientists ought to have some bearing on the way they think about the process of scientists changing beliefs.

"In reality, scientists do not proceed to appraise claims by explicit application of Bayesian methods. They do not, for example, report results by reporting their posterior probability assignments to one hypothesis compared with others." (Mayo 1996, pg. 89)

While we would not wish to dismiss Bayesian models of belief formation in markets, we do accept the basic thrust of Mayo's argument. If we, as scientists, habitually use hypothesis testing to help form our own beliefs, why is this not acknowledged when we think about how agents, in general, form beliefs?[2]

We consider a belief formation algorithm based on a Neyman-Pearson hypothesis test and call this *Inferential Expectations* (IE).[3] We assume that when a belief is overturned agents switch to RE.[4] Thus, RE is a special case of IE if agents are unconcerned about mistakenly changing their beliefs (the test size α equals unity). By linking RE to IE, we ground our expectations theory ultimately in the structure of the model, which provides modeling discipline. Furthermore, α becomes a

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[1] Near rationality has been applied to the failure of UIP in Gruen and Menzies (1995) and more recently in Bacchetta and van Wincoop (2006). Econometric learning has been applied to the failure of UIP in Chakraborty and Evans (2006).

[2] One author received criticism from a referee of a top-tier journal because significance was accorded to a test in the submitted paper with a p-value of 0.052.

[3] For a comprehensive discussion of IE with an application to the foreign exchange market and supporting empirical evidence see Menzies and Zizzo (2005, 2007). For a concise overview of IE, highlighting the range of applications see Henckel et al. (2007).

[4] An implication of this is that Bayesian methods and IE need not conflict. Agents could be infrequent Bayesians, with the time interval between updates determined by a hypothesis test.

metric for rationality. If $\alpha = 0$, agents are completely unresponsive to evidence, while if $\alpha = 1$ they make the best possible use of evidence.

Our idea is closest in spirit to Goldberg and Frydman (1996) and Frydman and Goldberg (2003), who allow agents to conduct hypothesis tests over models. Their program, in turn, can be traced back to a nascent discussion by Rappaport (1985). IE is also related to Foster and Peyton Young's (2003) game-theoretical work on hypothesis testing by bounded-rational agents on their opponents' repeated games strategies. It can be considered as a 'fast and frugal heuristic' (Gigerenzer et al., 1999) of belief formation characterized by information-gathering and information processing costs. A final link to the existing literature can be made by considering econometric learning (Evans and Honkapohja 2001). Learning, like IE, gains some legitimacy from the practice of economists, and is actually a sub-case of IE. If we take account of both the recursive least-squares estimates *and their standard errors*, then regression-based IE with $\alpha = 1$ becomes econometric learning.

The theory of IE lends itself to a myriad applications.[5] In what follows we demonstrate the power of IE in the context of a standard model of time inconsistency in monetary policy. This application is simple and elegant and is supposed to pave the way for more comprehensive uses of IE.

3. The Standard Barro-Gordon Model

This section reviews the basic Barro-Gordon model (1983b). Even though the analysis is well known, derivation of the key equations will prove useful when comparing it to the analysis under inferential expectations. We will focus in particular on studying the range of central bank inflation announcements that are enforceable (time consistent) when the private sector has rational expectations.

3.1. Model Setup

There are two players, the monetary authority and homogeneous private agents (firms and households). The monetary authority directly controls the inflation rate π_t in each period in effort to minimize the following intertemporal loss function:

$$V_t = \sum_{s=0}^{\infty} q^s L_s \tag{1}$$

[5] See Henckel et al. (2007).

where the period loss function is given by

$$L_s = \frac{a}{2}\pi_s^2 - b\left(\pi_s - \pi_s^e\right), \qquad a, b > 0; \tag{2}$$

and q denotes a constant discount factor. It takes private agents' inflation expectations π_{t+i}^e, $i = 0, 1, 2...$, as given. Since there are no lags in the system the current choice of inflation imposes no direct constraints on future choices, viz. the optimization problem becomes static.[6] The private sector acts rationally and thus tries to form the best possible estimate of the inflation rate π.

The period loss function is standard—the first term captures the nonlinear cost of a deviation of inflation from its target, which is normalized to zero. The second term models the benefit from unanticipated inflation as encapsulated in a Lucas supply function. The constant parameters a and b describe the monetary authority's relative weights for inflation stabilization and output expansion, respectively.

3.2. Discretionary Policy

Minimizing L_t, taking π_t^e as given, delivers the time consistent inflation rate,

$$\pi_t^d = \frac{b}{a}, \tag{3}$$

where the d-superscript denotes "discretion".

Under rational expectations, $\pi_t^e = \pi_t^d = b/a$. The period t loss to the monetary authority is therefore given by

$$L_t^d = \frac{b^2}{2a}. \tag{4}$$

3.3. Policy Under a Rule

If the policy maker can commit to an announced inflation rate, π^*, and the private sector believes this announcement, private agents' expectations become endogenous, implying $\pi_t^e = \pi_t = \pi^*$. Under this scenario the second term in the loss function (2) vanishes and the inflation rate minimizing the objective function is

$$\pi_t^c = \pi^* = 0 \tag{5}$$

[6] At the start of period t the monetary authority and the private sector share the same information set.

where the c-superscript denotes "commitment". The cost to the monetary authority is given by

$$L_t^c = 0. \tag{6}$$

Clearly, the loss under discretion is greather than under commitment, replicating the conventional result that rules are better than discretion.

3.4. Cheating

If private agents believe an announced rule, the policy maker has an incentive to systematically deceive (cheat) agents and create surprise inflation. Assume the monetary authority announced an inflation target of zero which is believed by the private sector, viz. $\pi^e = \pi^* = 0$. The monetary authority chooses π_t that minimizes its loss function:

$$\pi_t^{ch} = \frac{b}{a}, \tag{7}$$

where the ch-superscript denotes "cheating". The associated cost to the monetary authority is given by

$$L_t^{ch} = -\frac{b^2}{2a}, \tag{8}$$

which is actually a net gain since the loss is negative. This differs from the discretionary case (4) because agents are deceived whilst believing the rule ($\pi^e = 0$ rather than $\pi^e = b/a$).

3.5. Sustainable Equilibria

Discretion is the most costly outcome for the policy maker, followed by a policy rule, followed by cheating after a credible announcement. That is,

$$L_t^d < L_t^c < L_t^{ch}. \tag{9}$$

For a policy rule to be credible (believed) the temptation (benefit) must be less than or equal to the enforcement (cost, loss) or else private agents will know the monetary authority will cheat. In other words, the rule must be time consistent. The ideal rule from society's point of view ($\pi^* = 0$) is not time consistent. The following sections compute the range of announcements for which the benefits of cheating are less than the cost, and this range does not include zero.

3.5.1. Trigger Strategy

Assume that the private sector follows a simple trigger strategy. In period $t-1$, if expected inflation equals actual inflation, then expected inflation in period t continues to equal the announced rate. If actual inflation deviates from expected inflation, agents assume the monetary authority has cheated and inflation expectations are revised to b/a:

$$\pi_t^e = \begin{cases} \pi^* & \text{if} \quad \pi_{t-1} = \pi_{t-1}^e \\ \pi_t^d & \text{if} \quad \pi_{t-1} \neq \pi_{t-1}^e \end{cases} \tag{10}$$

Employing a trigger strategy in the above model reduces it to a two-period problem. If the monetary authority does not cheat, retaining its reputation, inflation and the utility loss will remain at zero. If the monetary authority cheats it will enjoy a utility gain in period t and suffer a loss in period $t+1$. The model reverts to the discretionary outcome because agents are rational and have adjusted their inflation expectations to b/a. In period $t+2$, as $\pi_{t-1} = \pi_{t-1}^e$, the private sector once again expects inflation to equal the announced target, $\pi_t^e = \pi^*$. The monetary authority will therefore cheat if the benefits in period t exceed the present value of the loss incurred in period $t+1$.

3.5.2. Benefits and Costs

The temptation (=benefit) to cheat in period t is the difference between the utility loss of a policy under a rule and a policy associated with cheating. For a given target inflation rate π^* the benefit to cheating is given by

$$L_t^c - L_t^{ch} = \frac{a}{2}\left(\frac{b}{a} - \pi^*\right)^2. \tag{11}$$

The greater the distance between π^* and b/a, the greater the temptation to cheat. At $\pi^* = b/a$, there is no temptation to cheat because b/a is already the optimal inflation rate under discretion, that is, the monetary authority would choose b/a anyway. As $|b/a - \pi^*|$ rises, the temptation to cheat increases as the policy maker is farther and farther away from its optimal inflation rate b/a. Thus, the *temptation curve* looks like a parabola.

The benefit of adhering to the announced policy rule ("enforcement") equals the present value of the cost of cheating in period $t+1$. For a given target inflation rate π^*, this cost is equal to the cost of reverting to discretionary policy in $t+1$, discounted by q,

$$q\left[L_t^d - L_t^c\right] = q\left[\frac{a}{2}\left(\left(\frac{b}{a}\right)^2 - (\pi^*)^2\right)\right]. \tag{12}$$

For values of π^* below b/a, in increase in π^* reduced the costs of cheating at an increasing rate. When $\pi^* = b/a$, the policy rule cannot be enforced because the cheating value of inflation, π^{ch}, is equal to expected inflation, π^e. When $\pi^* > b/a$, the benefit to enforcement becomes negative as the policy maker prefers the punishment of reverting to b/a. Thus, the *enforcement curve* is an inverted parabola.

3.5.3. Best Enforceable Rule

For a rule to be enforceable the benefit to cheating (temptation) must be less than or equal to the cost of cheating (enforcement).[7]

Figure 1 shows the temptation and enforcement curves, with the costs and benefits on the y-axis and the announced inflation rule on the x-axis. At the ideal rule ($\pi^* = 0$), where the temptation and enforcement curves intersect the y-axis, the temptation curve lies above the enforcement curve.[8] Rational agents would know it was in the monetary authority's interest to cheat and hence would not believe a zero inflation rule. In principle, any π^* that lies in the interval

$$\left[\frac{b\,(1-q)}{a\,(1+q)}, \frac{b}{a}\right],$$

is enforceable. However, the policy maker will pick the value from within the enforceable range that minimizes its loss. This value is the lower bound of the enforceable range.

3.6. Conclusion

The Barro-Gordon model reveals that it is optimal for a monetary authority to follow a rule than to act discretely. Under discretion the monetary authority takes private sector expectations as given, whereas under a rule it takes them

[7]Note that private agents are assumed to have complete information, viz. they know temptation and enforcement equations.

[8]At $\pi^* = 0$, the benefit of cheating, $b^2/2a$, exceeds the cost of cheating, in present value terms, $q\left(b^2/2a\right)$.

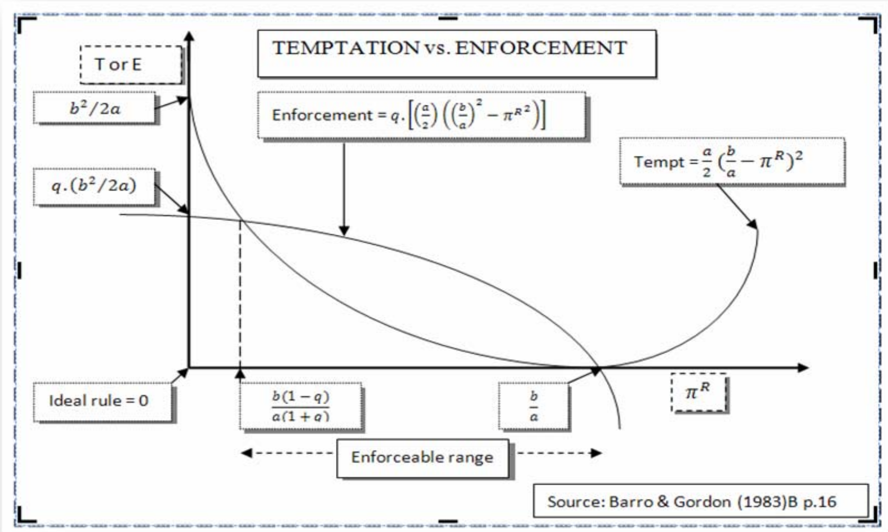

Figure 1: Range of enforceable equilibria in the original Barro-Gordon model

into account. Greater credibility and more transparency serve to heighten the monetary authority's reputation and its ability to successfully adopt a low inflation target.

The ideal rule of zero inflation is not time-consistent; only those inflation rates that lie in the enforceable range are. Without reputation the only credible inflation rate is b/a. Thus, reputation raises the costs of deviating from an announced target, delivering a superior inflation outcome (Taylor 1983).

4. The Barro-Gordon Model Under Inferential Expectations (Mark 1)

In their seminal article, Barro-Gordon (1983b) write:

> "We can modify the model so that punishments take place occasionally. For example, suppose that inflation depends partly on the policy maker's actions and partly on uncontrollable events. Further

assume that people cannot fully sort out these two influences on inflation, even ex post. Then people adopt a form of control rule where the policy maker loses reputation if the observed inflation rate exceeds some critical value." [pp. 26-27]

Barro-Gordon's proposed modification is elegantly captured by the assumption of inferential expectations. We feel that this kind of setup is much closer to reality. Most central banks are given an inflation target *range* and those central banks that are given a specific target without a range acknowledge that inflation will regularly deviate from its official target.[9] Target ranges are an admission that inflation is uncertain and not perfectly controllable.

In an uncertain world cheating could take place for several periods, as agents need time to decipher whether the monetary authority has changed its policy or whether observed inflation is merely noisy. To keep things simple, we only adjust the benefit to cheating (temptation) and leave the cost (enforcement) unchanged. If the benefit increases, rational agents will know the monetary authority has more incentive to cheat and will therefore tolerate a smaller band of inflation announcements.

The elements of the hypothesis test associated with inferential expectations in this model are:

1. The cognitive target is the monetary authority's announced inflation rate (π^*). Due to noise in the inflation process (events beyond the control of the monetary authority) agents must determine whether the cognitive target is being met by conducting the following hypothesis test:

2. H_0: the monetary authority is obeying the rule, there is no change in underlying inflation ($\pi^e = \pi^*$);
   H_1: the monetary authority has cheated ($\pi^e = \pi^{ch}$);

3. The signal is the number of periods for which $\pi > \pi^*$;

4. The test statistic is t, based on a sampling distribution to be discussed later.

[9] In the next section we take a closer look at central bank inflation targets.

4.1. Benefits and Costs

The benefit to cheating in period t is

$$\sum_{i=0}^{n} q^i \left(L_{t+i}^c - L_{t+i}^{ch}\right) = AQ, \tag{13}$$

where n denotes the period in which private agents decide the monetary authority is cheating and A and Q are defined as[10]

$$\begin{aligned} A &\equiv \frac{a}{2}\left(\frac{b}{a} - \pi^*\right)^2, \\ Q &\equiv \frac{1-q^n}{1-q}. \end{aligned}$$

The benefit of adhering to the announced policy rule is now given by

$$q^n \left[L_{t+n}^d - L_{t+n}^c\right] = q^n \left[\frac{a}{2}\left(\left(\frac{b}{a}\right)^2 - (\pi^*)^2\right)\right], \tag{14}$$

which is simply the cost, in present value of terms, of reverting to discretionary policy in period n.

4.2. Best Enforceable Rule

Once again, the boundaries of the range of enforceable (time consistent) inflation rates are determined by the points where the temptation and enforcement curves intersect. Setting (13) equal to (14) and solving for both roots of π^*, yields the upper bound (π_1^*) and the lower bound (π_2^*) of the range:

$$\left[\frac{b\left(Q-q^n\right)}{a\left(Q+q^n\right)}, \frac{b}{a}\right].$$

This is depicted in Figure 2.

Comparing this model to the original Barro-Gordon model reveals that the upper bound of the enforceable range, $\pi_1^* = b/a$, is the same in both cases, while π_2^* in the IE-model is a function of n, the number of periods the monetary authority can cheat for. The y-intercept of the temptation curve $\left(Q\left(b^2/2a\right)\right)$ is increasing in n since $\partial Q/\partial n > 0$. The y-intercept of the enforcement curve $\left(q^n\left(b^2/2a\right)\right)$ is decreasing in n. Hence, as the number of periods during which the monetary authority can cheat for increases, the enforceable range shrinks.

[10] Note that $L_{t+i}^c - L_{t+i}^{ch} = L_t^c - L_t^{ch}$, $i \neq 0$.

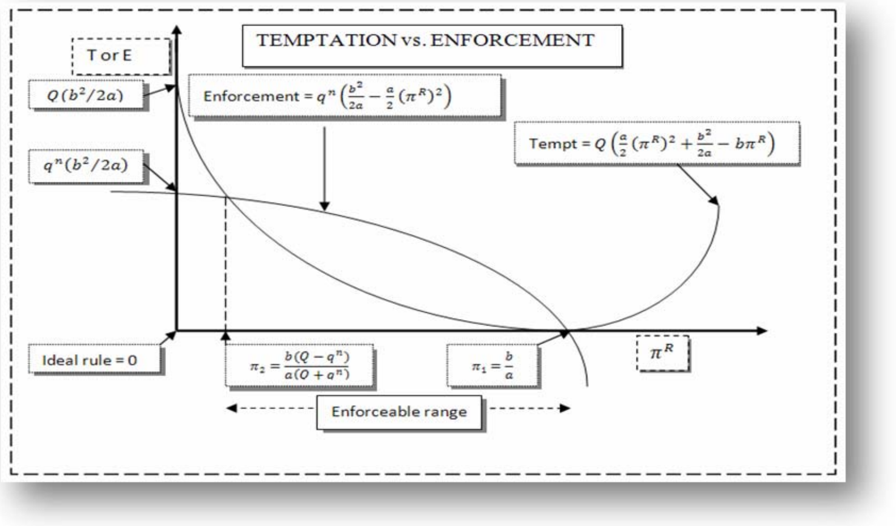

Figure 2: Range of enforceable equilibria in the Barro-Gordon model with inferential expectations

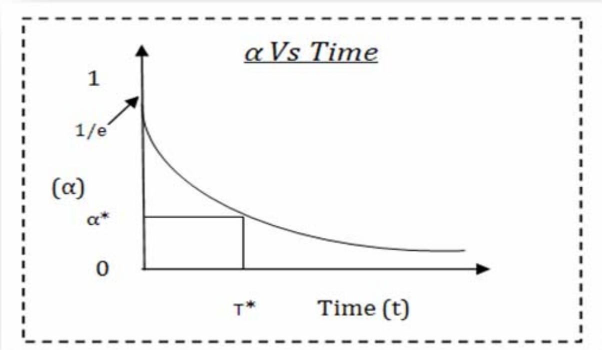

Figure 3: Maximum number of periods monetary authority can cheat

4.3. Inferential Expectations and Maximum Length of Cheating

Assume that the number of periods the monetary authority can cheat for is an exogenously determined exponential function of agents' conservatism, which is represented by their alpha test score. Thus, time is a function of ■. When agents are conservative they require a large amount of information to overturn their existing belief. In this case alpha test scores are low (0.01 or 0.05) and they need to observe actual inflation above the target rate for many periods before they reject the null hypothesis and conclude the central bank has cheated.

When observed inflation deviates from the official target, the probability that the monetary authority has *not* cheated decreases over time. This is captured in Figure 3 which shows the sampling distribution of t, the probability, at time t, that π deviates from π^* when H_0 is true.

More formally, the probability density function is given by

$$f(t) = e^{-t}$$

and the p-value by

$$p = \int_n^\infty f(t)\,dt = \int_n^\infty e^{-t}dt = \left[-e^{-t}\right]_n^\infty = e^{-n}.$$

Hence, H_0 is rejected if

$$p = e^{-n} < \alpha,$$

which, when solved for n, becomes

$$n > -\ln \alpha.$$

Evidently, the lower bound of the enforceable range and consequently the range itself becomes a function of a (agents' degree of conservatism):

$$\pi_2^* = \frac{b\left(Q - q^{-\ln \alpha}\right)}{a\left(Q + q^{-\ln \alpha}\right)}$$

with

$$Q = \frac{1 - q^{-\ln \alpha}}{1 - q}.$$

4.4. Comparison with Original Barro-Gordon

As the maximum number of periods the central bank can cheat for increases, the enforceable range decreases. The intuition is straightforward. If agents can be deceived for longer than one period, the temptation to cheat grows while the punishment for cheating remains the same. Rational agents know the temptation is greater and will therefore tolerate a smaller enforceable range.

Two extremes are useful to consider. First, consider the case where n approaches infinity. The lower bound of the range is thus

$$\pi_2^* = \lim_{n\to\infty} \frac{b\left(Q - q^n\right)}{a\left(Q + q^n\right)} = \lim_{n\to\infty} \frac{b\left(\lim_{n\to\infty} Q - 0\right)}{a\left(\lim_{n\to\infty} Q + 0\right)} = \frac{b}{a} = \pi_1^*.$$

Hence, the enforceable range collapses to zero and the only enforceable inflation rate is b/a.

Now consider the case where $n = 1$. In this case the lower bound is

$$\pi_2^* = \frac{b\left(Q - q^n\right)}{a\left(Q + q^n\right)} = \frac{b\left(1 - q\right)}{a\left(1 + q\right)},$$

which simply equals the lower bound in the original Barro-Gordon model. The latter model is therefore nested in the model with inferential expectations.[11]

[11] When $n = 1$, $\alpha = e^{-1} = 0.368$. In other words, if agents have very little conservatism in their beliefs and hold test sizes of $\alpha > 0.368$, the monetary authority is unable to cheat for longer than one period.

4.5. Discussion

When agents have inferential expectations in the Barro-Gordon model the enforceable range of equilibrium inflation rates falls. Because the inflation process is noisy, agents are unable to decipher whether an increase in observed inflation is due to noise or the central bank cheating. If agents are very conservative they require a large amount of information (many observations) before they change their beliefs. The monetary authority is thus able to cheat for several periods before being 'caught' and so the temptation to cheat grows. The private sector recognizes this and responds by tolerating a smaller range of inflation targets. Consequently, the enforceable range depends on the alpha value agents hold in their hypothesis tests. The original Barro-Gordon model is a special case of the model with inferential expectations in which the alpha value exceeds 0.368.

The major weakness of the Mark 1 model is that the bands are only a function of time and the exogenous degree of conservatism. In reality, the magnitude of the cheating must play a big part in determining how long a monetary authority can cheat for. More specifically, magnitude and time are inversely related. The monetary authority can cheat for a long time if it cheats by a small amount; cheating by a large amount implies it is likely to be found out much faster. A new inferential test statistic must be introduced.

5. The Barro-Gordon Model Under Inferential Expectations (Mark 2)

To measure both magnitude and time, assume that agents use a test statistic much the same as a standard Z statistic:

$$Z = \frac{\sum\left(\pi - \pi^*\right)/n}{\sqrt{\sum\left(\pi - \overline{\pi}\right)^2/n - 1}/\sqrt{n}},$$

where π denotes observed inflation, π^* the official inflation target, $\overline{\pi}$ the mean of observed inflation, and n the period in which the statistic is computed. The numerator of the above expression measures the average difference between the observed inflation rate and the monetary authority's inflation target, giving an indication of the magnitude of the cheat. The denominator is the standard error of the observations, viz. the standard deviation divided by the square root of n.

The elements of the hypothesis test associated with this test statistic are:

1. The cognitive target is the monetary authority's announced inflation rate (π^*);

2. Agents perform the following hypothesis test:
H_0: the monetary authority is obeying the rule, there is no change in underlying inflation ($\pi^e = \pi^*$);
H_1: the monetary authority has cheated ($\pi^e = \pi^{ch}$);

3. The signal is the Z score;

4. The test statistic is the critical Z score, Z^*, based on a normal distribution, determined by agents' alpha values.[12]

If the test statistic is greater than the critical Z score, the private agents reject the null hypothesis and conclude the monetary authority has cheated. Agents' alpha sizes are exogenous, with higher alpha values implying less belief conservatism.

Assume the official inflation target $\pi^* = 0$. Due to the noise inherent in the inflation process the monetary authority announces that actual inflation may fluctuate within a target range of -0.5 to 0.5. The true inflation process is given by

$$\pi = \pi^* + \varepsilon,$$

where $\varepsilon \sim N(0, \sigma^2)$. This setup is not dissimilar from current central bank practice. Given the official inflation target range, agents can recover the standard deviation of the inflation process.[13] Under this assumption the Z statistic simplifies to

$$Z = \frac{\sum (\pi - \pi^*)}{\sigma \sqrt{n}}.$$

One can easily see that an increase in the magnitude of the cheat ($|\pi - \pi^*|$) leads to a larger test statistic, as does an increase in the length of the cheating period.

Assuming agents have inferential expectations and standard deviation calculations come from the announced inflation target range, the monetary authority can calculate the magnitude of the cheat in each period so as to keep the test statistic

[12] As always, Student's t distribution should be used for small samples.

[13] In the current example, if agents have an alpha value of 0.05, the target range roughly equals two standard deviations around the target π^*.

just below the critical value. Formally, equate the Z statistic to the critical value Z^* and solve for π_n:

$$Z = \frac{\sum(\pi - \pi^*)}{\sigma\sqrt{n}} = Z^*_{\alpha/2}$$

$$\Rightarrow \pi_n = Z^*_{\alpha/2}\sigma\sqrt{n} + n\pi^* - \sum_{i=1}^{n-1}\pi_i.$$

In this way the monetary authority could cheat for several periods while retaining its reputation.

Table 1 shows the magnitude of the cheat in each period $(\pi - \pi^*)$ to keep agents' test statistic exactly at the critical value.

n	$\boldsymbol{\sigma}$	$\boldsymbol{\sigma}\sqrt{n}$	$\boldsymbol{\pi} - \boldsymbol{\pi}^*$	$\sum(\pi - \pi^*)$	$\mathbf{Z}^*_{\alpha/2}$
1	0.25	0.25	0.49	0.49	1.96
2	0.25	0.35	0.20	0.69	1.96
3	0.25	0.43	0.16	0.85	1.96
4	0.25	0.50	0.13	0.98	1.96
5	0.25	0.56	0.12	1.10	1.96
6	0.25	0.61	0.10	1.20	1.96
7	0.25	0.66	0.10	1.30	1.96
8	0.25	0.71	0.09	1.39	1.96
9	0.25	0.75	0.08	1.47	1.96
10	0.25	0.79	0.08	1.55	1.96

Table 1: Magnitude of cheat in each period to retain credibility

If central banks were interested in increasing employment temporarily they could do so by increasing the inflation rate to just under the critical value shown above. This can occur without agents changing their inflation expectations, provided the cheat becomes smaller in size as time progresses. Alternatively, suppose there was a positive inflation shock and political factors prevented the monetary authority from raising interest rates straight away, inflation stabilization could be postponed as long as the rise in inflation did not push the test statistic past the critical value.

5.1. Discussion: Is Announcing a Target Range Important?

Most central banks around the world follow a range or a specific target for inflation. Table 2 summarizes the targeting regimes for eight OECD countries. Those that do follow a specific target acknowledge that inflation fluctuates around this target, within an acceptable range.[14] The Norges Bank (2007) acknowledges that the average rate has been 2% over the past 5-10 years. The SNB (2007) specifies inflation must stay below 2% per year, creating a range of 0-2% with emphasis towards the upper end. The notable exception is the U.S. Federal Reserve which does not specify a range at all but advocates price stability as one of its multiple policy mandates. Does this mean the U.S. does not follow a monetary policy rule even though it there is wide consensus that a rule is optimal?

Country	Central Bank	Range	Values	Notes
Australia	RBA	yes	2-3%	over the medium term
New Zealand	RBNZ	yes	1-3%	over the medium term
USA	Fed Reserve	no	none	mandate includes full
				employment, stable prices &
				moderate long term interest rates
Canada	BoC	yes	1-3%	target is 2%
UK	BoE	yes	1-3%	target is 2%
Euro Area	ECB	no	2%	target is below but close to 2%
				over the medium term
Norway	Norges Bank	no	2.5%	over the medium term
Switzerland	SNB	no	2%	inflation not to exceed 2%
Source: central bank websites				

Table 2: Inflation targets for 8 selected countries

An inflation band or target does not in itself represent a policy rule. It provides a target but does not tell us how to get there (Brash 1998, p. 6). The rules vs. discretion literature currently distinguishes between two types of policy rule: instrument rules and targeting rules. As described by Bernanke (2004), instrument

[14] Bernanke (2007c) aptly summarized this as follows: "The only economic forecast in which I have complete confidence is that the economy will not evolve along the precise path implied by our projections."

rules (feedback policy rules) require the central bank to set the overnight interest rate in response to the behavior of a relatively small number of macroeconomic variables. A Taylor rule falls into this category. Forecast targeting (or, more generally, forecast-based policy) rules require the central bank to predict how the economy will respond to alternative plans in the medium term (6-24 months) using economic models. Such targeting rules are applied at the Bank of England, Sweden's Riksbank and Norges Bank.

The U.S. is the only OECD country not to adopt a specific process or target, McCallum (2004) argues:

> "The absence of rule-based policymaking means the absence of any systematic process that the public can understand and use as the basis for its expectations about future policy. The Fed apparently sees communication as a device for affecting expectations, but rational private agents form expectations on the basis of their understanding of the process by which the central bank actually conducts policy. If the central bank fails to adopt a process involving rule-based policymaking—that is, a commitment to some clearly stated objectives—its attempts to influence expectations are unlikely to be productive." [p. 370]

The profession is split over which of the two approaches is more successful. McCallum & Nelson (2005) argue that instrument rules are superior as they are more transparent. Forecast-based policies typically rely on complex economic models which the private sector is unlikely to understand. Moreover, targeting rules run the risk of model misspecification: if the model used for forecasting is incorrect, policy prescriptions are likely to be incorrect also. Under a regime of inflation forecast targeting a central bank has more discretion, which can be problematic if the wrong people are making the decisions.

Svensson (2005) and an increasing number of his colleagues argue that targeting rules are superior. "A (specific) targeting rule specifies a condition to be fulfilled by the central bank's target variables (or forecasts thereof)." [p. 614] The theoretical reasons against instrument rules and in favor of targeting rules are:[15] (1) an instrument rule will be suboptimal if some important variables are omitted; (2) instrument rules do not leave room for judgmental adjustments; (3) commitment to an instrument rule does not leave room for adjustment if new information arrives; (4) instrument rules do not capture existing monetary policy

[15] See, for example, McCallum and Nelson (2005).

well. Bernanke (2004) emphasizes that under a targeting rule communication is of utmost importance to control expectations as targeting rules involve complicated models which agents do not understand sufficiently well.

Bernanke (2007c) recently addressed the issue of transparency and communication, announcing that the Federal Reserve will henceforth release the FOMC members' economic projections every quarter. The projections, extending to a three-year horizon, will apply to overall inflation, core inflation, real GDP and the unemployment rate. The publication will include a comparison with previous projections, charts showing central tendencies and ranges for each variable ("fan charts"), and the distribution of members' projections. A discussion by each member on the major forces shaping the outlook and the risks to the outlook will also be released. The first of these publications was published on 20 November 2007.

In practice most central banks use a combination of the two types of rules, although forecast-based policy seems to dominate.[16] Brash (1998, p.9) is more forceful when he states that, "in practice a binding commitment to a single rule is simply not feasible because not enough is known about the structure of the economy or the shocks it will face." This conviction confirms Greenspan's justification for the lack of a formal instrument rule at the Federal Reserve. (Greenspan 1997, p. 1)

How does the theory of inferential expectations contribute to this debate?

The theory of inferential expectations makes no claim about the superiority of instrument rules or targeting rules; the issue there is over the accuracy of the former and the transparency of the latter. IE does suggest that, by announcing a target range, agents forming hypothesis tests can easily infer a standard deviation. The tighter the range, the smaller the standard deviation implied, hence the smaller the magnitude and duration the central bank can cheat for. This seems to imply that all central banks should increase their ranges without bound without fear of detection. However, while the public may accept the monetary authorities' judgment of the standard deviation for plausible values, it is likely to turn to other sources as the range becomes infinitely wide.

The problem associated with a larger range and standard deviation is that agents will start to expect higher average inflation rates. A standard mean-variance analysis implies that an increase in the variance leads to an increase in the risk premium, assuming investors are risk averse. In this context an increase in the range (larger standard deviation) leads to a higher premium demanded by workers when negotiating wages and by firms when setting prices. That is,

[16] See Bernanke (2004) and Svensson (2005).

if an inflation target was 2-3%, workers and employers, when determining wages and prices can be fairly certain inflation will be close to 2.5%. If the inflation band was 0-5% workers and firms need to be compensated for the increased uncertainty, which is likely to generate an inflationary bias. Thus a larger range is suboptimal. If the central bank announces no range or target agents face the problem of an unknown standard deviation (this is explained in the next section) and they will require even higher premiums as they do not have a satisfactory way to measure whether or not the central bank has cheated. A range which allows for some flexibility but is not too large to generate a significant inflationary bias is optimal.[17]

The absence of a clearly defined inflation target range in the U.S. means the Fed is following a targeting rule without a specific target. According to the theory of inferential expectations agents therefore cannot infer a standard deviation for their hypothesis tests. Bernanke (2007c) argues the reason why the Federal Reserve does not use a specific inflation target is because it has the dual objective of full employment and price stability. For agents with inferential expectations this is beside the point. The Federal Reserve runs a policy of "trust our judgment and we will communicate our policies." This trust is arguably more fragile than the trust bestowed upon central banks who operate under legislated inflation targets.

The recent announcement by the Federal Reserve to improve communication is a step in the right direction and should ossify this trust. Moreover, Bernanke (2007c) states that, while other central banks advocate price stability only, they practice "flexible" inflation targeting, taking into account economic growth, employment and financial stability. Flexible inflation targeting does not seem that different from U.S. policy, yet other central banks specify inflation targets or ranges.

The issue is ultimately an empirical one but the theory of inferential expectations suggests that specifying an inflation range is desirable for monetary policy.

6. Further Issues

6.1. Z Statistic and Outliers

Until now we have assumed that agents knew the population standard deviation, σ, and used this number in their hypothesis tests. It makes more sense to assume

[17] A formal analysis of this will be presented in later versions of this paper.

that agents do not know the population standard deviation and use the sample standard deviation instead. This, however, poses some potential problems.

We have seen that the test statistic Z increases when n increases. That is, the longer observed inflation remains away from the mean, the more likely the actual mean differs from the hypothesized mean, and the null hypothesis would be rejected.

However, counter-intuitive results are possible with one-off large cheats; in particular, it is possible that the test statistic reverts to 1 as $(\pi - \pi^*) \to \infty$. To see why this is possible, rewrite the Z statistic as

$$Z = \left(\frac{1}{\sqrt{\frac{n}{n-1}}} \right) \frac{\sum (\pi - \pi^*)}{\sqrt{\sum (\pi - \overline{\pi})^2}}.$$

Ignore the term in parantheses and focus on the other term which includes the cheat (or magnitude) effect, $(\pi - \pi^*)$. Suppose that past observations were in a small range (say between -0.5 to 0.5) so that the sum of the cheats $\sum (\pi - \pi^*)$ is some small number close to zero. The denominator $\sqrt{\sum (\pi - \overline{\pi})^2}$ will also be a small number. If a big cheat now occurs (or a random draw from a distant tail of the distribution), the numerator increases by approximately the same amount as the denominator, and so the test statistic reverts back to 1.

If the test statistic was initially above one the increased cheat would generate a fall in the statistic; if it was initially below one the test statistic would rise. This is undesirable for if the magnitude of the cheat increases (the observation is moving further away from the mean), there should be greater confidence that the observed inflation rates do not equal the mean. The null hypothesis should be rejected and the test statistic should increase. One possible way around this is to assume that the calculation of the sample standard deviation itself is subject to inferential expectations, its value only updated periodically.

6.2. Credibility

All central banks accept that credibility is crucial for managing inflationary expectations. New Zealand in the 1990s amended its central bank charter to allow for a dismissal of the central bank governor should inflation move outside the target range (Romer 2006, p.519). The Bank of England Act stipulates that if inflation moves outside the 1-3% target range, “the Governor of the Bank must write an open letter to the Chancellor explaining the reasons why inflation has increased or fallen to such an extent and what the Bank proposes to do to ensure inflation comes back to the target" (Bank of England 2007). The benefit of increased credibility is better control of inflation expectations: agents will more likely believe the central bank is following a rule if inflation does happen to deviate from target. When inflation in New Zealand moved outside the band in 1996, "the debate was not about that we were outside the band, it was about why we were outside the band" (Brash 1998, p.8). This episode suggests that the RBNZ enjoyed enough credibility to let inflation remain outside the band for some time without a change in expectations.

The IE model suggests that credibility plays an important role in determining whether a central bank has cheated. Assume the monetary authority cheats by the same amount in each period so that $(\pi - \pi^*)$ equals a constant. Write the Z statistic as

$$Z = \frac{\sum (\pi - \pi^*)}{\sigma \sqrt{n}} = \frac{n_{start-cheat} (\pi - \pi^*)}{\sigma \sqrt{n_{start-history}}}.$$

When the monetary authority is following a rule $E[\pi - \pi^*] = 0$. The number of periods in the numerator $n_{start-cheat}$ only begins counting once the cheating starts. The number of periods in the denominator $n_{start-history}$ refers to the number of periods since the model began. Thus the longer the monetary authority does not cheat $(\pi = \pi^*)$, the more credibility it gains. The denominator increases and so, for the Z statistic to reach the critical value, a larger or longer cheat is necessary compared to having no credibility and starting the cheat in the initial period $(n_{start-cheat} = 1)$.

7. Conclusion

Economics differs from the natural sciences because expectations about the future crucially matter. The importance of expectations is succinctly captured in the Barro-Gordon model of time inconsistency in monetary policy.

Recent discomfort with pervasive assumption of rational expectations has led to several alternative hypotheses of expectations formation. This paper takes the idea of inferential expectations (IE), as proposed by Menzies and Zizzo (2005), and applies it to the Barro-Gordon model. The simple version of the model showed that, by introducing IE into the Barro-Gordon model, the enforceable range of equilibrium inflation rates shrinks. The more conservative agents are, the smaller the enforceable range becomes. This model has the weakness of not distinguishing between magnitude and size of the cheat. The Mark 2 model is more realistic and complete and could easily be applied to other economic problems. The model shows that when the monetary authority knows how agents form their beliefs, it can cheat and exploit the Phillips curve for some time without being ‘caught’. The model also shows that announcing a target *range* is optimal in any monetary policy framework and that credibility is desirable.

Inferential expectations describe how agents form their beliefs in periods of uncertainty. If there is no uncertainty in an IE model, agents’ hold rational expectations. In reality the world is uncertain and variables which being forecast are noisy. For example, all central banks concede inflation cannot be perfectly predicted and will vary around a target.

Inferential expectations has other realistic properties. It explains why variables tend to be subject to periods of inertia interspersed with discrete shifts, rather than continuously updated, as implied by rational expectations. In most periods agents underuse marginal pieces of information but, at the point of rejection, they overuse marginal pieces of information (Menzies and Zizzo 2005, p. 4). This idea of inertia is a good description of many economic processes, including inflation. Galbraith (1997, p. 98) point out, “the empirical evidence is in almost uniform agreement that inflation is highly inertial.”

The idea of inferential expectations seems to have a strong theoretical grounding. This paper showed some of the implications of inferential expectations for monetary policy. The theory enables a coherent *joint* analysis of inflation targets and ranges and the associated optimal policy, which is left for future work.

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