Welfare Improving Coordination of Fiscal and Monetary Policy

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Abstract

The paper considers a simple model in which monetary and fiscal policies are formally independent, but still interdependent - through spillovers of macroeconomic outcomes. It shows that the average equilibrium levels of inflation, deficit, debt, and output depend on the two policies' (i) potency (elasticity of output with respect to the policy instruments); (ii) ambition (the level of their output target); and (iii) conservatism (the degree of inflation aversion). However, it is the relative degrees of these that matter rather than the absolute degrees for each policy. Therefore, coordination of monetary and fiscal policy is found to be superior to non-cooperative Nash behaviour for both policymakers. Interestingly though, it is coordination in terms of the policies' ambition, rather than conservatism, that is essential. Furthermore, ambition-coordination may be welfare improving even if the policymakers' objectives are idiosyncratic, and their coordinated output targets differ from the socially optimal one.

Keywords: coordination, interaction, monetary policy, fiscal policy, central bank, government, inflation, deficit, debt.

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1. Introduction

This paper shows that taking both monetary ($M$) and fiscal ($F$) policy into account in the examination of the optimal setting of each policy may be of crucial importance. It demonstrates that doing so may qualify or even reverse some conventional results.

The need to study the effects of $M$ and $F$ policy interaction were first highlighted by the seminal work of Tinbergen (1954) and Mundell (1962). Unlike most of the subsequent literature that examined the direct interaction\footnote{That is the ability of the government (fiscal policymaker) to affect monetary policy outcomes through the appointment of the central banker (Rogoff (1985)), optimal contract with the central banker (Walsh (1995)), through overriding the central banker (Lohmann (1992)), or through a mixture of these, Hughes Hallett and Libich (2007).} this paper follows in the footsteps of Sargent and Wallace (1981) and Dixit and Lambertini (2003) by focusing on the indirect interaction. Specifically, we examine how $M$ and $F$ policies affect each other through spillovers of their outcomes. As a matter of experimental control, we will separate the direct and indirect effects by assuming that the policymakers are fully independent (in a constitutional and political sense) and have perfect control over their instruments - the level of inflation for $M$ and the growth rate of nominal debt (size of deficit) for $F$. These assumptions are arguably realistic in the current era of independent central banks that are quite capable of achieving a certain desired level of inflation on average, even in the presence of shocks.

Using a parsimonious non-controversial model, a simple extension of Barro and Gordon (1983), we focus on long run macroeconomic outcomes - in both levels and growth rates. These are the first order effects. But there are also second order stabilization consequences, which show up in the variability of the outcomes around the long-run trend in the presence of shocks. These second order effects are considered in Section 4.

2. Model

2.1. Setup. Our basic model is a simple extension of Barro and Gordon (1983). The Lucas supply relationship summarizes the economy and is extended to also include the effect of fiscal policy (we will not use the time subscript as our interest will lie in the one shot game)

$$x = \mu (\pi - \pi^e) + \rho g,$$

where $x, \pi, \pi^e$, and $g$ denote the output gap, inflation, inflation expected by the public, and the growth rate of real debt respectively.\footnote{Note that this specification ensures money neutrality. Section 4 considers some extensions to the supply curve, including shocks, and shows that are main findings are unchanged.} The parameters $\mu$ and $\rho$ are positive and denote the potency of $M$ and $F$ policy respectively. Let us define the growth rate of real debt in the standard fashion

$$g = G - \pi,$$

where $G$ is the growth rate of nominal debt (which can also be thought of as the size of the deficit, with $G = 0$ expressing a balanced budget). $G$ and $\pi$ are assumed to be the instruments of $M$ and $F$ policy, independently set and perfectly controlled.
The policymakers’ one period utility function follows the convention in the literature and can, as Woodford (2003) has shown, be derived from microfoundations\(^5\)

\[
u^i = -\beta^i (x - x_T^i)^2 - \pi^2,
\]

where \(i \in \{M, F\}\) is the set of players and the inflation target of both policies is set to zero. Further, \(\beta^i > 0\) denotes the degree of policy conservatism (lower \(\beta^i\) values denoting greater conservatism, as in Rogooff (1985)). Finally, \(x_T^i \in \mathbb{R}\) denotes the level of the output gap target, which we will refer to as the degree of ambition.

We will distinguish the policymakers according to three criteria. First, following Rogooff (1985) we will refer to \(\beta^M < \beta^F\) and \(\beta^M \geq \beta^F\) as the cases of conservative and liberal central banker respectively. Second, from the point of view of the natural rate hypothesis we will refer to those with \(x_T^i = 0\), \(x_T^i > 0\), and \(x_T^i < 0\) as responsible, over-ambitious, and under-ambitious respectively. Third, from a political economy viewpoint we will refer to the governments with \(x_T^F = \bar{x}_T\) and \(x_T^F \neq \bar{x}_T\) as benevolent and idiosyncratic respectively, where \(\bar{x}_T \in \mathbb{R}\) is the level of the socially optimal output gap target (all variables referring to social welfare will be denoted by a bar)\(^6\).

For simplicity, we will assume that \(F\) ambition is the only aspect in which the social welfare function \(u\) may differ from the government’s own utility function \(u^F\). Further, it is assumed that the public, like the policymakers, has complete information and rational expectations. Finally, all parties can move every period. These standard assumptions will enable us to focus on the policy interaction as there will be no reputational issues\(^7\).

2.2. Solution. Focusing on the one shot simultaneous game we have, using (1)-(3) and rational expectations, the following reaction functions

\[
\pi = \frac{\beta^M (\rho - \mu)(\rho G - x_T^M)}{1 + \beta^M \rho (\rho - \mu)} \quad \text{and} \quad G = \frac{x_T^F}{\rho} + \pi.
\]

Solving jointly yields the following equilibrium outcomes (denoted by a star throughout)

\[
\pi^* = \beta^M (\rho - \mu)(x_T^F - x_T^M) \quad \text{and} \quad G^* = \frac{x_T^F}{\rho} + \beta^M (\rho - \mu)(x_T^F - x_T^M), \quad \text{and}
\]

\[
g^* = \frac{x_T^F}{\rho} \quad \text{and} \quad x^* = x_T^F.
\]

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\(^5\)The players can be thought of as discounting the future with \(\delta_M\) and \(\delta_F\) being their discount factors. But as we will be focusing on the one-shot game, this will not play any role in the analysis.

\(^6\)Note that we do not impose a specific value for \(\bar{x}_T\). This setup nests four main cases of interest in which the government is: (i) benevolently over-ambitious, \(x_T^F = \bar{x}_T > 0\), (ii) idiosyncratically over-ambitious, \(0 < x_T^F \neq \bar{x}_T\), (iii) benevolently responsible, \(x_T^F = \bar{x}_T = 0\), and (iv) idiosyncratically responsible, \(x_T^F = 0 \neq \bar{x}_T\). The main reason for \(x_T^F \neq \bar{x}_T\) identified in the literature has been the presence of various political economy features such as naïve voters; the desire to offset a central bank that is too conservative; or to overcome the effects of distortionary taxation or monopolistic competition which prevent markets clearing at full employment.

\(^7\)For the alternative cases in which (i) reputations matter or (ii) the players’ actions feature some rigidity and cannot be reconsidered every period (due to costly wage bargaining, information processing, and/or commitment), see Hughes Hallett and Libich (2007) and Libich, Hughes Hallett, and Stehlík (2007) respectively.
where (5) reports the nominal variables (policy instruments) $\pi$ and $G$ and (6) reports the real variables $g$ and $x$.

3. Results

This section reports results for our baseline model. For the sake of simplicity, we first derive results for four macroeconomic variables $\{\pi, G, g, x\}$ in terms of their average or equilibrium levels. Section 3 then introduces shocks and shows that our earlier propositions still hold in a stochastic environment.

3.1. Monetary and Fiscal Policy Interdependence. This section demonstrates that the two policies may be interrelated even if they are formally independent.

**Proposition 1.** For almost all parameter values, (i) the equilibrium value of the $M$ policy instrument is not only a function of $M$ policy conservatism, ambition, and potency, but also of the ambition and potency of $F$ policy; and (ii) the equilibrium value of the $F$ policy instrument is not only a function of $F$ policy ambition and potency, but also determined by the conservatism, ambition, and potency of $M$ policy.

**Proof.** Inspection of (5) reveals that both $\pi^*$ and $G^*$ are functions of $\beta^M, x^F_T, x^M_T, \rho$ and $\mu$ for all $\rho \neq \mu$ and $x^F_T \neq x^M_T$.

The proposition implies that the other policy’s setting generally matters.

**Proposition 2.** The inflation target and balanced budget are time-inconsistent for almost all parameter values, including those in which the respective policymaker is responsible and/or conservative.

**Proof.** It follows from (5) that $\pi^* \neq 0$ and $G^* \neq 0$ (unless $\rho = \mu$ or $x^F_T = x^M_T$ for the former and $\mu = \frac{\beta^M, \rho^2(x^F_T - x^M_T) - x^F_T}{\beta^M, \rho(x^F_T - x^M_T)}$ for the latter). This means that the levels $\pi_t = 0$ and $G = 0$ are not the equilibrium choices of the respective policymaker and hence they are time-inconsistent in the sense of Kydland and Prescott (1977).

**Proposition 3.** Not only the magnitude, but also the direction of the central banker’s best response to a budgetary outcome depends on the relative potency of the two policies. There exist circumstances under which the optimal response to a fiscal expansion (deficit) is:

(i) $M$ tightening (and this can be true even for an over-ambitious $M$),
(ii) $M$ easing (and this can be true even for an under-ambitious or responsible $M$),
(iii) no change in the stance of $M$ policy.

Put differently, $\pi$ can be a strategic substitute to $G$, a strategic complement to $G$, or independent of $G$.

**Proof.** Inspection of the reaction functions in (4) reveals that $\pi^*$ is (i) decreasing in $G$ iff $\rho \in (\max\{\mu - \frac{1}{\rho^2}, 0\}, \mu)$, (ii) increasing in $G$ iff $\rho \in (0, \mu - \frac{1}{\rho^2}) \cup (\mu, \infty)$, and (iii) independent of $G$ iff $\rho = \mu$.

We need to stress that the difference in optimal $M$ responses is not due to business cycle fluctuations - to control for these we have made our baseline model deterministic. Instead, it is due to the feedback effect between $M$ and $F$ policy. Specifically, the level
of inflation affects the optimal size of the deficit, which in turn affects the optimal level of inflation. Since the respective magnitudes of these effects may differ, the direction of the monetary responses (i.e., whether $M$ counter-acts or accommodates) may too.

Also note that in this setting, $G$ is always a strategic substitute to $\pi$, i.e., $G^*$ is increasing in $\pi_t$ for all parameter values. In Section 4, it will be shown that in the presence of shocks this is not always the case.

3.2. Monetary Policy Outcomes. Next we examine the optimal setting and outcomes of the $M$ policy instruments.

Proposition 4. Inflation (deflation) bias can occur even if either one or both policymakers are under-ambitious (over-ambitious).

Proof. It is claimed that parameter values exist under which $\pi^* > 0$ even if $x^M_T < 0$ and/or $x^F_T < 0$, and $\pi^* < 0$ even if $x^M_T > 0$ and/or $x^F_T > 0$. This can be seen in (5) - the former obtains under either $\rho > \mu$ and $x^F_T > x^M_T$, or $\rho < \mu$ and $x^F_T < x^M_T$; and the latter under either $\rho > \mu$ and $x^F_T < x^M_T$, or $\rho < \mu$ and $x^F_T > x^M_T$.

This is in contrast to the conventional view that high inflation is necessarily due to the excessive output target of the central banker and the government.

The following two propositions show the conventional wisdom of the Barro-Gordon literature - that $M$ policy conservatism and ambition have a monotone effect on the level of inflation - can also become reversed as a result of $M$ and $F$ interactions.

Proposition 5. The inflation target may be time-consistent and credible even if $M$ is over-ambitious and/or liberal.

Proof. The proposition claims that we may obtain $\pi^* = 0$ even if $x^M_T > 0$ and/or $\beta^M > \beta^F$. From (5) it follows that this will be the case iff $\mu = \rho$ or $x^M_T = x^F_T$. 

Remark 1. Proposition 5 shows that achieving the inflation target is not sufficient for us to conclude that the central bank is responsible and/or conservative - even in the absence of shocks.

Intuitively, since different combinations of parameter values provide the right incentives to deliver the inflation target, it may be impossible to infer the type of the central banker. For example, if $M$ and $F$ policy are equally potent, then neither $M$ nor $F$ policy preferences affect equilibrium inflation: under $\mu = \rho$ the level of $\pi^*$ is independent of $\{\beta^M, \beta^F, x^M_T, x^F_T\}$. Similarly, if the policymakers are equally ambitious, the time-inconsistency problem is alleviated.

Proposition 6. The direction (as well as the magnitude) of the effect of $M$ policy conservatism on the level of inflation depends on 1) the relative degree of $M$ and $F$ policy ambition and 2) the relative potency of $M$ and $F$ policy. Specifically, a more conservative central banker will:

(i) decrease inflation iff $M$ policy is either more potent and less ambitious than $F$ policy; or less potent and more ambitious than $F$ policy,

(ii) increase inflation iff $M$ policy is either both more potent and more ambitious than $F$ policy; or both less potent and less ambitious than $F$ policy,

(iii) not affect inflation iff $M$ and $F$ policy are equally potent and/or equally ambitious.
Proof. Equation (5) shows that (i) under $\mu > \rho \wedge x_M^T < x_F^T$ or $\mu < \rho \wedge x_M^T > x_F^T$ the value of $\pi^*$ is increasing in $\beta^M$; (ii) under $\mu > \rho \wedge x_M^T > x_F^T$ or $\mu < \rho \wedge x_M^T < x_F^T$ the value of $\pi^*$ is decreasing in $\beta^M$; and (iii) under $\mu = \rho \vee x_M^T = x_F^T$ the value of $\pi^*$ is not a function of $\beta^M$. \hfill $\Box$

Arguably, while it is less likely that $x_M^T > x_F^T$ since $M$’s ambition is commonly driven by $F$’s ambition, the case of $M$ policy being less potent than $F$ policy, $\mu < \rho$, may be plausible and hence all three claims may obtain (unlike in the Barro-Gordon literature in which only claim (i) obtains). Intuitively, a low inflation policy may stimulate the economy better than a high inflation policy - by increasing the value of real debt and hence magnifying the expansionary effect of $F$ policy better than any inflation surprise could.

Proposition 7. The direction (as well as the magnitude) of the effect of $M$ policy ambition on the level of inflation depends on the relative potency of $M$ and $F$ policy. But it does not depend on the degree of either $M$ or $F$ policy conservatism. Specifically, a more ambitious central banker will:

(i) increase inflation iff $M$ policy is more potent than $F$ policy,
(ii) decrease inflation iff $M$ policy is less potent than $F$ policy,
(iii) not affect inflation iff $M$ and $F$ policy are equally potent.

Proof. Equation (5) shows that (i) under $\mu < \mu$ the level of $\pi^*$ is increasing in $x_M^T$; (ii) under $\mu > \mu$ the level of $\pi^*$ is decreasing in $x_M^T$; and (iii) under $\mu = \mu$ the level of $\pi^*$ is not a function of $x_M^T$. \hfill $\Box$

It is important to consider why an under-ambitious (or responsible) $M$ policymaker might find it optimal to inflate. It is because, by doing so, he attempts to decrease the real value of the debt in order to reduce the expansionary effect of the $F$ policy and thus stabilize output closer to his target.

This result, like Proposition 4, alerts us to the fact that, in predicting macroeconomic outcomes, policy ambition has to be viewed in light of the other policy’s ambition. Specifically, the effects of an over-ambitious $M$ in the presence of an over-ambitious or under-ambitious fiscal policies may differ dramatically.

3.3. Fiscal Policy Outcomes. Let us now turn to examining the optimal outcomes of fiscal policy.

Proposition 8. Nominal debt may be increasing (decreasing) over time - through persistent deficits (surpluses) - even if either one or both policymakers are under-ambitious (over-ambitious).

Proof. It is stated that we can have $G^* > 0$ even if $x_F^T < 0$ and/or $x_M^T < 0$; and $G^* < 0$ even if $x_F^T > 0$ and/or $x_M^T > 0$. Rearranging of (5) shows that the latter obtains, inter alia, if

$$x_M^T < x_F^T \left(1 - \frac{1}{\rho \beta^M (\mu - \rho)}\right),$$

Note that the proposition qualifies the standard Barro-Gordon-Rogoff result by showing that, even under $\mu > \rho$, a conservative central banker may increase inflation.
where $\mu > \rho$. The former happens if the inequality in (7) is reversed.

This result may seem surprising - we commonly think of excessive spending and nominal debt accumulation as an outcome of policymakers with over-ambitious targets, and budget surpluses as those with under-ambitious or responsible ones. Intuitively, an under-ambitious or responsible $F$ policymaker may find it optimal to run deficits in an attempt to increase the real value of the debt and hence offset the (what they perceive as still too contractionary) effect of $M$ policy (note that $x^M_F < x^F_F$ in (7)). This achieves output closer to their target.

**Proposition 9.** The budget deficit and the growth rate of the nominal debt may be zero, even if $F$ is over-ambitious and $F$ policy is more potent than $M$ policy.

**Proof.** The proposition claims that we may obtain $G^* = 0$ even if $x^F_F > 0$ and $\rho > \mu$. From (5) it follows that this will be the case iff

$$\beta^M = \frac{x^F_F}{\rho (\rho - \mu)(x^M_F - x^F_F)},$$

which completes the proof.

**Remark 2.** Proposition 9 shows that we cannot conclude caution in concluding that any government that runs balanced budgets or surpluses is responsible. This is because the central banker may indirectly (through incentives) ‘discipline’ an otherwise over-ambitious $F$ policymaker.

Three things should be noted. First, the disciplining central banker is not necessarily conservative - $\beta^M$ in (5) may be greater or less than $\beta^F$. Second, the disciplining central banker is also not necessarily responsible - the $x^M_F$ value in (8) may also be positive or negative. Third, the price for this discipline is a deflationary $M$ policy.

The following proposition however shows that under some circumstances, the central banker can, depending on the relative potency and ambition in the $M$ and $F$ policies, accentuate the ambition of the $F$ policymaker and worsen the budgetary outcomes.

**Proposition 10.** An appointment of a conservative and/or less ambitious central banker may increase, decrease, or have no effect on the size of the budget deficit and the growth rate of the nominal debt.

**Proof.** It is claimed that $G^*$ may be decreasing, increasing, or independent of $\beta^M$ and/or $x^M_F$. In terms of $\beta^M$, equation (5) reveals that this is indeed the case under $(\rho - \mu)(x^F_F - x^M_F) < 0$, $(\rho - \mu)(x^F_F - x^M_F) > 0$, and $(\rho - \mu)(x^F_F - x^M_F) = 0$ respectively. In terms of $x^M_F$, equation (5) reveals that this is the case under $\rho > \mu$, $\rho < \mu$, and $\rho = \mu$ respectively.

This proposition is an analog of Propositions 6 and 7 in the realm of $F$ policy. Intuitively, whether a conservative and responsible central banker improves or worsens the
budgetary outcomes depends on what type of $F$ opponent he is facing. The opponent type determines how strongly and in which direction the opponent is expected to react.

**Proposition 11.** An election of a less ambitious government may decrease, increase, or have no effect on the size of the budget deficit and the growth rate of the nominal debt.

**Proof.** It is claimed that $G^*$ may be decreasing, increasing, or independent of $x_T^F$. Rearranging equation (5) reveals that this is indeed the case under $\mu < \tilde{\mu}(\rho), \mu > \tilde{\mu}(\rho)$, and $\mu = \tilde{\mu}(\rho)$ respectively, where the threshold $\tilde{\mu}(\rho)$ is the following

$$\tilde{\mu} = \rho + \frac{1}{\rho \beta M} > \rho,$$

which completes the proof.

This implies that caution should be exercised in concluding that a less ambitious government will surely improve the country’s fiscal position. But since the first case obtains for a larger parameter space (from the fact that $\tilde{\mu} > \rho$ it follows that it includes values of $\rho > \mu, \rho < \mu$, as well as $\rho = \mu$), it might be argued that a less ambitious government leads to an improvement in fiscal discipline more often than not.

### 3.4. Real Variables

This section examines how the setting of the two policies determines the size of real debt and output.

**Proposition 12.** Real debt may be increasing (decreasing) even if the nominal debt is decreasing (increasing); that is even if the government is running surpluses (deficits).

**Proof.** The proposition argues that we may obtain $g^* > 0$ even under $G^* < 0$; and $g^* < 0$ even under $G^* > 0$. Inspection of (5) tells us that the former is true if (7) is satisfied together with $x_T^F > 0$, and the latter is true if the inequality is reversed and $x_T^F < 0$. 

Since the price level may be moving in the opposite direction to the nominal debt in equation (5) meaning surpluses may be accompanied by positive inflation or deficits by deflation, the conventional positive correlation between the size of the deficit and real debt may be reversed.

**Proposition 13.** Real output may be persistently below (above) the natural level, even if the government is running deficits (surpluses).

**Proof.** It is stated that we can have $x^* < 0$ even if $G^* > 0$, and $x^* > 0$ even if $G^* < 0$. Inspection of (5) and (6) tells us that the latter is true if (7) is satisfied together with $x_T^F > 0$, and the former is true if the inequality in (7) is reversed and $x_T^F < 0$.

Interestingly, this Keynesian type of result obtains despite the fact that the supply function does not feature non-neutrality. Furthermore, the $x^* > 0$ case can be achieved by running surpluses, i.e., the government’s intertemporal budget constraint is met. Nevertheless, since the central bank is running significant deflation, the price to pay is accumulation of real debt and hence an accumulation of domestic or international imbalances.

**Proposition 14.** Under almost all parameter values, the degrees of M policy conservatism and ambition affect the levels of the deficit and nominal debt, but do not affect the level of the real debt.
Proof. Inspection of (4) reveals that for all $\mu \neq \rho$ the levels of $\pi^*$ and $G^*$ are functions of $\beta^M$ and $x^M_T$. In contrast, (5) shows that $g^*$ is not a function of these variables.

This is in contrast to the conventional wisdom in which, in the absence of indirect policy interaction, central bank conservatism affects the size of real debt through inflation, but not the size of the deficit and nominal debt that is set independently by the government. In other words, the effect of $\beta^M$ and $x^M_T$ coming through $M$ and $F$ policy may cancel each other out.

**Proposition 15.** Even if $M$ policy is more potent than $F$ policy, and even if it influences inflation, the size of the deficit, and nominal debt through its conservatism and/or ambition, it may have no effect on real output.

Proof. Inspection of (6) shows that even if $\rho < \mu$ and $\pi^*$ and $G^*$ are functions of the monetary policy parameters $\beta^M$ and $x^M_T$, $x^*$ is not a function of these parameters. □

Also note that there is an asymmetry in the effect of the two policies; $F$ policy has more impact on $M$ policy than the other way round. This is because its impact is not dependent on surprising the public. While standard, this specification may be too ‘favourable’ to what $F$ policy can actually do. Section 4.2 therefore considers a case in which a deficit may actually have, even without the contribution of $M$ policy, a contractionary rather than an expansionary effect. Further, Section ?? postulates a case in which one or both policymakers are averse to the variability of real debt.

The following two propositions further highlight the dichotomy between nominal and real variables.

**Proposition 16.** Even if the relative potency of $M$ and $F$ policy only affects the nominal variables, $\pi$ and $G$, and not the real variables, $g$ and $x$, it still affects the utility of both policymakers and social welfare.

Proof. If $\pi^*$ is a function of $\mu$ and $\rho$ (which (5) shows to be for all values except $x^M_T = x^F_T$), then it follows from (3) the players’ and the society’s utility will be affected, and this is regardless of whether $\mu$ and $\rho$ affect $g^*$ and $x^*$. □

**Proposition 17.** (i) The degree of $F$ policy conservatism affects no nominal or real variables in the model.

(ii) The degree of $F$ policy ambition affects all nominal and real variables in the model for almost all parameter values.

Proof. Inspection of (5) and (6) reveals that while $\pi^*, G^*, g^*$ and $x^*$ are all functions of $x^F_T$ (with the exception of $\mu = \rho$ for $\pi^*$), none of them is a function of $\beta^F$.

The first claim may seem surprising in light of Proposition 6 which showed that $M$ policy conservatism affected the levels of the deficit and nominal debt for almost all parameter values. The second claim suggests that it is the $F$ policy’s output gap target level that plays a crucial role in determining the macroeconomic outcomes of both policies. In contrast to that, the relative weight that $F$ places on the deviations from this target does not have any importance. This suggests that $x^F_T$ may be the variable with the greatest overall macroeconomic impact and the one to focus on. Unfortunately, while the determinants of the government’s ambition have received a great deal of research
attention (in the political business cycle literature), its effect on the outcomes of \( M \) policy has not been explored in detail.

3.5. Optimal \( M \) and \( F \) Policy Coordination. The above results suggest that the relative degrees of \( M \) and \( F \) policy conservatism, ambition, and potency may play a more important role in macroeconomic outcomes than their levels do individually. In particular, they show that policymakers are often trying to offset the behaviour of the other policymaker which may lead to undesirable outcomes for both the policies and society. Therefore, two specific questions need to be addressed:

1) How conservative and ambitious should \( M \) and \( F \) policy be: what is the socially optimal institutional setting for these policies with respect to \( x^M_T, x^F_T, \beta^M \) and \( \beta^F \)?

2) Is this socially optimal scenario incentive compatible in a non-coordinated game in which the respective policymakers choose their policy parameters independently, or is policy coordination required? If so, is this only if the policymakers have idiosyncratic objectives or even if they are benevolent? Further, is it \('ambition-coordination', 'conservatism-coordination',\) or both that can improve social welfare?

To examine these questions let us extend our game by endogenizing the four policy variables \( \{x^M_T, x^F_T, \beta^M, \beta^F\} \). At the beginning of the game, in period \( t = 0 \), the corresponding policymakers choose simultaneously the values of their \( x^M_T \) and \( \beta^M \) (this timing is not crucial to the results obtained). Let us assume that all chosen \( \{x^M_T, x^F_T, \beta^M, \beta^F\} \) can be observed by the opponent as well as the public before the period 1 simultaneous move of \( \pi, \pi^e, \) and \( G \) is made.

**Proposition 18.** There exists infinitely many pure strategy Nash equilibria in the endogenous game, all such that \( \{x^M_T = x^F_T \in \mathbb{R}, \beta^M, \beta^F \geq 0, \pi, G^* \} \). They all yield the same, highest attainable utility to both policymakers, which is strictly greater than that of any non-Nash outcomes (pure or mixed). Since they all are some combination of \( x^M_T \) and \( x^F_T \), ambition-coordination is optimal for both policymakers.

**Proof.** It is shown in Appendix A that all pure strategy Nash equilibria are such that \( x^M_T = x^F_T \in \mathbb{R} \), and \( \pi^* \) and \( G^* \) satisfy (5) - with any levels of \( \beta^M \) and \( \beta^F \). In those Nash equilibria we therefore have \( \pi^* = 0 \) and \( x^* = x^F_T \) from (6), and hence \( u^i = 0, \forall i \) from (3), which is the unique maximum of \( u^i \). From the fact that (i) there exist an infinite number of pure strategy Nash equilibrium ambition levels, (ii) both policymakers are indifferent between them but prefer them to all non-Nash levels, and (iii) there exist no beliefs that would indicate any \( x_T^M \) of the opponent to be more likely, it follows that the non-coordinated values of \( x^M_T \) and \( x^F_T \) will almost certainly differ from each other and therefore deviate from the pure strategy Nash equilibria (from a game theoretic perspective the most 'likely' non-coordinated outcome is the Nash in mixed strategies). The fact that any \( x^M_T \neq x^F_T \) is inefficient yielding \( u^i < 0, \forall i \) completes the proof.

Note that \( M \) and \( F \) policy ambition are complements. For demonstration see Figure 1.

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10Since we know that the optimal value of \( \beta^M \) may depend on the size of shocks (Rogoff (1985)), we will need to revisit this result in the next section where stochastic disturbances are introduced.
Proposition 19. There exists a unique socially optimal combination of the degrees of M and F policy ambition, \( x^M_T = x^F_T = \bar{x}_T \). It belongs to the set of pure strategy Nash equilibria and is incentive compatible. Nevertheless, ambition-coordination may improve social welfare even if the policymakers are idiosyncratic, and coordinate on a certain combination of output targets other than the socially optimal one. In contrast, conservatism-coordination affects neither the utility of the policymakers nor social welfare.

Proof. See Appendix B.

Intuitively, coordination of M and F policy on the level of their output targets is desirable for both players as well as the society because a specific combination of \( x^M_T \) and \( x^F_T \) delivers the optimal inflation target, \( \pi^* = 0 \). Since M policy cannot affect the equilibrium level of average output (Proposition 15), this is the best long-run M policy setting. Thus ambition-coordination is socially optimal even if the policymakers are not benevolent and coordinate on an idiosyncratic output target.\(^{11}\)

The case in which ambition-coordination is not socially optimal is one in which coordination moves the output target away from \( \bar{x}_T \). This could be the case if the central bank’s output target is (pre-set) at a level different from the socially optimal one, and

\(^{11}\)Economic theory identifies two main reasons for \( x^F_T \neq \bar{x}_T \). First, the presence of naïve voters and other political economy features often seem to lead governments to be excessively ambitious. Second, since \( x^F_T \) is affected by various F policy settings that are legislated - eg health, welfare and pension schemes - there may be significant persistence in the value of \( x^F_T \) across subsequent governments.
the central bank has greater ‘bargaining power’ in the ambition-coordination process than the benevolent government (with \(x_T^T = \bar{x}_T\)).

While Proposition 18 shows that any value of \(\beta^M\) constitutes a Nash equilibrium, and Proposition 19 argues that there is no need for conservatism-coordination, it may still be the case that the value of \(\beta^M\) matters. Since the socially optimal \(\beta^M\) value may be affected by shocks, see eg Rogoff (1985), we will examine it in the next section.

4. Extensions

This section considers two generalizations of our baseline model and shows that all the above results carry over.

4.1. Extension 1: Shocks. Let us incorporate a supply shock by augmenting (1):

\[
x = \mu(\pi - \pi^e) + \rho g + \varepsilon,
\]

where \(\varepsilon\) is a shock with a zero mean. For simplicity, we will provide the solution for the case of an i.i.d shock with variance \(\sigma^2\), but it will be apparent that all our conclusions will hold for various data generating processes including persistent shocks. As elsewhere in the literature, we will assume that the policymakers can observe the shock in real time. Nevertheless as a robustness check, we will consider a case in which this is not true for the public.

Symmetric Information. In this case the public, like the policymakers, can observe the current shock before forming expectations. The equilibrium macroeconomic outcomes analogous to (5)-(6) become the following (all are denoted by \(\varepsilon\) in the superscript)

\[
\pi^e = \beta^M (\rho - \mu)(x_T^F - x_T^M)
\]

and \(G^e = \frac{x_T^F - \varepsilon}{\rho} + \pi^e\),

\[
g^e = \frac{x_T^F - \varepsilon}{\rho}
\]

and \(x^e = x_T^F\).

Asymmetric Information. In this case the public, unlike the policymakers, cannot observe the current shock before forming expectations. The equilibrium macroeconomic outcomes analogous to (5)-(6) become the following (all are denoted by \(\varepsilon\) in the superscript)

\[
\pi^{ee} = \beta^M (\rho - \mu)(x_T^F - x_T^M)
\]

and \(G^{ee} = \pi^{ee} - \frac{1 + \beta^M \rho(\rho - \mu)}{\rho \left[ 1 + \beta^M (\rho - \mu)^2 \right]} \varepsilon\),

\[
g^{ee} = \frac{x_T^F}{\rho} - \frac{1 + \beta^M \rho(\rho - \mu)}{\rho \left[ 1 + \beta^M (\rho - \mu)^2 \right]} \varepsilon
\]

and \(x^{ee} = x_T^F\).

Let us first revisit our results reported above in the presence of shocks.

**Proposition 20.** The average levels of \(\{\pi, G, g, x\}\) in both the symmetric and asymmetric cases are equivalent to those of our baseline deterministic model in Section 3. Therefore, interpreting all references to \(\{\pi, G, g, x\}\) therein as average/long-run/trend levels, Propositions 1-19 hold even in the presence of shocks.
Proof. Comparison of (5)-(6) with (11)-(12) and (13)-(14) shows that the deterministic components of all four macroeconomic variables coincide across the three settings.

The following propositions report some additional results implied by (11)-(14).

**Proposition 21.** The stabilization (cyclical) outcomes of $\pi$ and $x$ are unaffected by neither policies’ ambition, conservatism, nor potency, and this is regardless of the public’s information about the shock, and even if both policies have optimally responded to the shock.

Proof. The inspection of (11)-(14) shows that the stochastic $\varepsilon$ components of $\pi$ and $x$ is not a function of $\{x^F_T, \beta^*, \mu, \rho\}$ in either the symmetric or asymmetric case. It is straightforward to check that both players’ reaction functions include the shock.\footnote{The $F$ and $M$ policy reaction functions are the following: $G^* = x^F_T - \frac{\varepsilon}{\rho} + \pi$ and $G^M = \frac{1 + \beta^M \rho(\rho - \mu)}{\rho [1 + \beta^M (\rho - \mu)^2]} [\pi(\rho - \mu) + x^F_T - \varepsilon] - \mu \beta^M (\rho - \mu)x^M_T$.}

This result, among others, calls the conventional conclusions of the Barro-Gordon model without $F$ policy in question. In the conventional model, the central banker’s private information necessarily leads to an improvement in stabilization as it may be exploited to surprise the public. In contrast, stabilization in the presence of fiscal policy is equivalent in the symmetric and asymmetric cases.

Let us now consider the optimal choice of $\beta^M$.

**Proposition 22.** Even in the presence of shocks, and regardless of whether or not the public can observe them in real time, it is socially optimal to appoint an ultra-conservative central banker, $\beta^M = 0$. Such extreme degree of conservatism is optimal not only for the central banker of any ambition (ie incentive compatible), but also for a government of any type (including those that are over-ambitious and strongly dislike output variability). The welfare benefits of this appointment increase in the degree of ambition-mis-coordination.

Proof. Let us start claim (i) by noting that an ultra-conservative central banker, $\beta^M = 0$, ensures inflation on target for all three scenarios, in all periods, and for any degree of the policies’ ambition and size of the shock. Formally, under $\beta^M = 0$ we have $\pi^* = \pi^*_t = \pi^*_T = 0, \forall t, x^F_T, i, \sigma^2_{\varepsilon}$ from (5), (11), and (13) respectively. Since the equilibrium levels of inflation and output in all three cases, $\{\pi^*, x^*, \pi^*_T, x^*_{T}, \varepsilon\}$, are independent of the shock, this increases the utility of both policymakers ($\forall x^F_T, i, \sigma^2_{\varepsilon}$) as it brings average inflation closer to the target without compromising stabilization outcomes or increasing the variability of inflation and output. Welfare benefits increase in the degree of ambition-mis-coordination is implied by the fact that under $x^F_T \neq x^M_T$ we have $|\pi^*|$ increasing - and $\bar{u}$ decreasing - in the difference $|x^F_T - x^M_T|$.

\footnote{The $F$ and $M$ policy reaction functions are the following: $G^* = x^F_T - \frac{\varepsilon}{\rho} + \pi$ and $G^M = \frac{1 + \beta^M \rho(\rho - \mu)}{\rho [1 + \beta^M (\rho - \mu)^2]} [\pi(\rho - \mu) + x^F_T - \varepsilon] - \mu \beta^M (\rho - \mu)x^M_T$. This also shows that, unlike in the deterministic and symmetric scenarios, under information asymmetry $G$ may also be a strategic complement to (or independent of) $\pi$.}
The optimality of $M = 0$ for all parties is natural in the deterministic case as there are no shocks to be stabilized. It is however somewhat surprising in the symmetric case, and very surprising in the asymmetric case in which the policymakers can exploit their private information about the shock. In both cases this is because in the presence of two policy instruments the shocks are stabilized more effectively than with $M$ policy alone.

Let us contrast the findings to those of Rogoff (1985). In his analysis (of the asymmetric case), it is socially optimal to appoint a conservative, but never an ultra-conservative central banker, $\beta_M > 0$, and/or 2) coordinate with a central banker (of any $\beta_M$) on their output targets (ambitions). Since the latter may be perceived as conflicting with central bank independence, and more difficult to do since the natural rate of output may vary over time, the former avenue may be preferable.

Remark 3. The government has two substitute (but not mutually exclusive) ways of maximizing social welfare and achieving credibility in the inflation target: 1) appoint an ultra-conservative central banker, $\beta_M = 0$, and/or 2) coordinate with a central banker (of any $\beta_M$) on their output targets (ambitions). Since the latter may be perceived as conflicting with central bank independence, and more difficult to do since the natural rate of output may vary over time, the former avenue may be preferable.

These findings seem to be consistent with the observed trends towards more independent and conservative $M$ policy around the globe during the past three decades and the emergence of explicit inflation targeting. Given that independent central bankers are commonly responsible, $x_M = 0$, it may be hard for the government to coordinate on an over-ambitious (politically driven) output target. Therefore, the government is more likely to use the first option; appoint an ultra-conservative central banker, and still pursue its over-ambitious output goals through $F$ policy. The fact that the majority of industrial countries (i) achieve their output targets with great precision, and (ii) still run budget deficits on average, is consistent with this interpretation.

4.2. Extension 2: Contractionary Deficits. As the presence of the shock was shown not to alter our results, Proposition 20 let us revert to our deterministic environment. Extend the Lucas supply function as follows

\begin{equation}
    x = \mu(\pi - \pi^e) + \rho g + \tau G,
\end{equation}

where the parameter $\tau \in \mathbb{R}$ expresses the potency of nominal-$F$ policy (as opposed to the potency of real-$F$ policy $\rho$). This enables us to better consider (i) a contractionary effect of a budget deficit and (ii) an asymmetry in the effect of fiscal policy. The equilibrium macroeconomic outcomes analogous to (5)-(6) become the following (all are denoted by $\tau$ in the superscript)

\begin{equation}
    \pi^\tau = \beta_M(\rho - \mu)(x_F^T - x_M^T) \quad \text{and} \quad G^\tau = \frac{x_F^T + \rho \beta_M(\rho - \mu)(x_F^T - x_M^T)}{\rho + \tau}.
\end{equation}

\begin{equation}
    g^\tau = \frac{x_F^T - \tau \beta_M(\rho - \mu)(x_F^T - x_M^T)}{\rho + \tau} \quad \text{and} \quad x^\tau = x_F^T.
\end{equation}

The following generalization is then implied.
Proposition 23. The equilibrium inflation and output levels under the generalized supply function (13) are equivalent to those of the deterministic, symmetric, and asymmetric scenarios. Therefore, all results of Propositions 1-19 regarding \( \pi \) and \( x \) hold for all \( \tau \in \mathbb{R} \).

Proof. By inspection of (16)-(17).

This shows that our main results are robust to the specification of \( F \) policy. Perhaps most surprisingly, the finding of Proposition 13 about output persistently deviating from the natural rate may still obtain. Specifically, an output level that is permanently above the natural rate, \( x^\tau > 0 \), can be achieved even under negative values of \( \tau \), i.e. even if expansionary \( F \) policy (deficits) have a contractionary effect. This is because in such case the \( F \) policymaker finds it optimal to run surpluses, \( G^\tau < 0 \), which have a stimulatory effect on the economy.

5. Summary and Conclusions

6. References

Appendix A. Proof of Proposition 18

Proof. Solving by backwards induction, the Nash equilibrium values of inflation and output are those derived in Section 3 under exogenous ambition and conservatism and reported in (5)-(6). Moving backwards, substitute these into the players’ utility functions and differentiate them with respect to the degree of \( M \) and \( F \) ambition respectively to obtain

\[
\frac{\partial u^M}{\partial x^M_T} = 2 [\beta^M(\rho - \mu)]^2 (x^F_T - x^M_T),
\]

\[
\frac{\partial u^F}{\partial x^F_T} = -2 [\beta^M(\rho - \mu)]^2 (x^F_T - x^M_T).
\]

Setting equal to zero and rearranging implies that the non-coordinated Nash equilibria obtain for any \( x^M_T = x^F_T \in \mathbb{R} \). For the rest of the proof see the main text.

Appendix B. Proof of Proposition 19

Proof. Substitute the equilibrium values from (5) and (6) into the social welfare function (which is (3) with \( F \) and \( x_T \)) and differentiate with respect to the degree of \( M \) and \( F \) ambition respectively to obtain

\[
\frac{\partial u^*}{\partial x^M_T} = 2 [\beta^M(\rho - \mu)]^2 (x^F_T - x^M_T),
\]

\[
\frac{\partial u^*}{\partial x^F_T} = -2 [\beta^M(\rho - \mu)]^2 (x^F_T - x^M_T).
\]

Setting equal to zero and rearranging implies that the unique social welfare maximum obtains under \( x^M_T = x^F_T = x_T \). As this is one of the pure strategy Nash equilibria reported in Proposition 18 and these deliver the highest attainable utility to both players, it is incentive compatible. Whether or not ambition-coordination improves social welfare when \( x_T^M = x_T^F = x_T \) depends on the chosen levels of \( x_T \) in its presence and absence.

The proof of Proposition 18 explained that there is no unique outcome in the former
case, and the same is true in the latter (even though the set of possible outcomes under ambition-coordination is smaller, it is still infinite). Nevertheless, it is obvious that there exist a range of circumstances under which ambition-coordination improves social welfare even if the coordinated ambition levels do not coincide with the socially optimal one. For example, the case in which policymakers coordinate on an output target that would have been chosen by \( F \) even in the absence of ambition-coordination. It then follows from (6) that (i) the output outcome of ambition-coordination and non-coordination are equivalent, but (ii) the inflation outcomes differ, in that under ambition-coordination we have \( \pi^* = 0 \) and without ambition-coordination we have \( \pi^* \neq 0 \). The proof of Proposition of [18] then implies \( \bar{u} = 0 \) and \( \bar{u} < 0 \) respectively.

In terms of conservatism-coordination, its redundancy is implied by the fact that \( \beta^C \) does not affect any macroeconomic variables, see Proposition [17(i)]. Therefore, the combination of \( \beta^M \) and \( \beta^C \) plays no role either.