A Small State-Space Model of the Australian Economy

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ABSTRACT

We estimated a small state space macroeconomic model using maximum likelihood and the Kalman filter to obtain joint estimates of the medium-run paths of natural rates for the Australian economy. Our unobserved component analysis indicates that in 2006, actual output was just above potential output, the normal rate of growth had fallen to a 20 year low point, unemployment was well below the natural rate presaging inflationary pressures, and that the real rate of interest was significantly below its natural rate, suggesting that monetary policy was possibly too expansionary. The imprecision of these estimated paths supports caution in policy design.

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1. Introduction

A traditional view of business cycles is that they are short-run stochastic movements of real variables around their smoothed trend values. These smoothed trends or natural rates play an important role in macroeconomic models as benchmarks to compare with current values. Wicksell (1936) introduced the concept of the natural interest rate, and more recently, there has been a revival in the literature on the subject following Woodford's (2003) book. In contemporary terms, the natural rate of interest is the equilibrium real rate (sometimes called the neutral rate) that would arise if wages and prices were completely flexible, given current factors. Phelps (1967) and Friedman (1968) introduced the related idea of the natural rate of unemployment. More specifically, the tradeoff between inflation and unemployment is temporary, so that the actual unemployment rate converges to the natural rate, at which point the inflation rate remains constant. Thus this benchmark unemployment rate is also known as the non-accelerating inflation rate of unemployment (NAIRU). When the economy is anchored at the NAIRU, GDP must be at the natural level of output, which is sometimes called the level of potential output. A short-run output gap emerges if GDP deviates from the natural level of output. However this potential output need not be constant in the medium run—given productivity growth and factor accumulation, there will be a normal rate of growth of potential output in the medium run. As the horizon progresses to the long run, this growth rate becomes the steady-state growth rate of the economy. Short-run deviations from the natural (and normal) rates in the medium run can be explained by the presence of imperfect information (e.g. Lucas, 1972) or nominal rigidities. These deviations affect movements in aggregate demand and supply, which in turn stimulate adjustment processes to return the economy to the medium-run equilibrium.

These natural rate and level concepts are central to the conduct of monetary policy. An inflation-targeting central bank needs to assess the level of economic activity variables in relation to their natural values to judge the pressures on inflation relative to its target and on any other target variables. When output grows faster than normal and exceeds its potential value, the unemployment rate will fall short of the NAIRU, wage inflation will rise, the real

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interest rate will be below its natural level, and so there will be upward pressure on inflation. The central bank is likely to tighten monetary policy to steer inflation and output back to their target and natural values. The short-term interest rate rises from its current position until the medium-run equilibrium is restored, and monetary policy returns to its neutral stance.

Although these natural economic indicators provide useful information to economists and policymakers, they are unobservable by nature and must be inferred from the data. The objective of this paper is to estimate a multivariate, unobserved components (UC) model for the Australian economy that allows for the simultaneous estimation of the paths of potential output and its normal growth rate, the NAIRU, the natural real interest rate, and a time preference factor. The multivariate state space model comprises a dynamic IS equation of the output gap representing aggregate demand (AD), an expectations-augmented Phillips curve that represents aggregate supply (AS), an Okun's relation connecting cyclical movements of output to unemployment, and a first-order condition from intertemporal optimization giving the medium-run relationship between the real interest rate and output growth. The model is estimated using maximum likelihood over the period 1984Q1 to 2006Q4, extracting unobservable state variables with the Kalman filter. Inflation information from the AS relation is used to infer the unemployment gap (defined as the difference between actual and natural unemployment), which in turn connects to the output gap (defined as the difference between actual and potential output) through an Okun's equation. The dynamic IS equation allows the real interest rate gap to exert influence on the product market, hence inferring the natural real rate of interest.

The paper proceeds as follows. In section 2, different univariate and multivariate measures of natural rates are discussed. This discussion motivates the multivariate model outlined in section 3. Section 4 describes the data, some econometric issues related to Kalman filter and presents the parameter estimates and the multivariate UC smoothed natural rates and their Monte Carlo simulated confidence intervals. Section 5 offers some concluding remarks.

2. Univariate and multivariate measures of natural rates

A widely used procedure to decompose macroeconomic variables (such as real output) into trend (or potential output) and cyclical (or the output gap) components is the Hodrick and Prescott (1997) filter.[1] The smoothness of the Hodrick-Prescott (or HP) stochastic trend depends on the input value of an *ad-hoc* smoothness parameter. If the value of the exogenous parameter is set to zero, the trend component and the actual series match each other; if the value of the parameter goes to infinity, the trend component approaches a linear deterministic trend. They recommended using the value of 1600 to work with quarterly data.[2] Baxter and King (1999) derived an estimate of the output cycle by passing the data through a filter that pre-specifies the relevant frequencies for the cycle and thus its persistence. Their approximate band-pass filter defines the cycle as having spectral power in the range between 6 and 32 quarters.

Pure statistical methods that simply 'let the data speak' do not include potentially useful information about the supply side of the economy and the business cycle contained in macroeconomic relationships such as the Phillips curve, Okun's law, and other indicators such as output capacity utilization.[3] Laxton and Tetlow (1992) proposed a multivariate extension to the univariate HP filter by conditioning the computation of time-varying potential output on additional economic relationships. Boone et al. (2000) applied a multivariate HP filter to derive the level of potential output for twenty one OECD countries. To estimate potential output for Australia, de Brouwer (1998) incorporated information from inflation, unemployment, and capacity utilization. Gruen et al. (2005) conditioned their estimates on the Phillips curve using real-time output data.

Another class of models—known as the unobserved components (UC) model—offers two advantages over the multivariate HP filter: (1) it permits a more complex system of

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[1] The original paper appeared in 1980 as a Carnegie-Mellon discussion paper, and was eventually published unchanged in 1997.
[2] Although there are methods available to choose the smoothness parameter optimally–where the mean of the squares of the differences of the estimated and true cyclical values is minimized–but this relies on the assumption that one knows precisely the data generating process.
[3] While there are many other univariate filters which may have better properties, our focus is on the gains from multivariate extensions.

dynamics; and (2) estimation is relatively more straightforward with the structural parameters estimated by maximum likelihood and using the Kalman filter to extract the time paths of the unobserved variables (or natural rates).[4]

Within the UC framework, several papers attempted to estimate natural rates using different macroeconomic relationships. Clark (1989) and Kuttner (1991) used an Okun's equation, which defines the level of (or change in) observable unemployment as a function of the unobservable output gap, to derive the level of U.S. potential output. Conditioned on the Phillips curve, Kuttner (1994) computed the level of potential output at which the economy maintains a constant rate of inflation. The constant-inflation natural unemployment rate (or the NAIRU) is provided by King et al. (1995), Staiger et al. (1996) and Laubach (2001) using a similar framework.

Instead of conducting partial analysis on potential output or the NAIRU, improvements can be made by estimating a system of equations that simultaneously features the Phillips curve, which imposes a constant-inflation restriction on the path of potential output or the NAIRU, and incorporates the covariation restrictions on cyclical output and cyclical unemployment through the Okun's relation. Some examples that model this mutual dependency include Apel and Jansson (1999a, 1999b) and Benes and N'Diaye (2004).

Movements from the real interest rate relative to its natural rate can also be embedded in the IS relation for the output gap describing product market equilibrium. Incorporating this extra channel is likely to enhance the estimation of the cyclical paths of unobservable variables in the economy.

The natural real interest rate is likely to vary over time, and in an intertemporally optimal setting will be determined by factors such as underlying productivity growth and the rate of time preference. For example, Laubach and Williams (2003) found substantial variations in the natural interest rate over the past four decades in the U.S. They also

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[4] The multivariate HP filter implemented by Laxton and Tetlow (1992) is a two-step procedure. First, the economic relationships are separately estimated. The regression residuals are inserted into the multivariate HP minimization problem to estimate the unobservable variable. This two-step procedure is repeated with several iterations until convergence is achieved. See Boone (2000) for more details.

suggested that there is an approximate one-for-one relationship between the natural rate of interest variation and changes in the growth rate of potential GDP.

3. A multivariate model of unobserved components

We begin our model with two identities where output and unemployment are decomposed into a stochastic trend and the stochastic cyclical variations around this trend. The trend components are taken to be the level of potential output and the natural rate of unemployment (or the NAIRU) that are associated with the medium-run equilibrium when prices and wages have fully adjusted to shocks. When demand or supply shocks occur, in the short run, deviations from the trend values are observed because of nominal rigidities, and these are defined as the output and unemployment gap measures.

$$y_t = y_t^* + \tilde{y}_t \tag{1}$$

$$u_t = u_t^* + \tilde{u}_t \tag{2}$$

In (1) and (2), y_t is the log of real GDP, y_t^* is the log of potential GDP and $\tilde{y}_t$ denotes the output gap; u_t is the unemployment rate, u_t^* stands for the value of the NAIRU, and $\tilde{u}_t$ represents the unemployment gap. Note that all variables are potentially time-dependent.

Following Rudebusch and Svensson (1999), the aggregate demand side of the economy is described by a reduced-form IS equation (3):

$$\tilde{y}_t = a_1 \tilde{y}_{t-1} + a_2 \left(r_{t-8} - r_{t-8}^* \right) + a_3 \Delta LTOT_{t-4} + a_4 \Delta \tilde{y}_t^{G7} + \varepsilon_t^{\tilde{y}} \tag{3}$$

A stationary AR(1) process is specified for the dynamic evolution of the output gap ($\tilde{y}_t$).[5] As in Laubach and Williams (2003), a real interest rate gap $(r_{t-j} - r_{t-j}^*)$ is included in the output gap equation. After preliminary OLS estimations using general to specific tests, we found that the 8th lag (2 years) of the real interest rate gap should be included. Stone et al. (2005) similarly found it necessary to include lags 1 to 7 of the same variable to uncover the effect of a change in monetary policy on the real economy. Rising terms of trade is expected

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[5] Watson (1986) and Clark (1987) for the U.S. economy and de Brouwer (1998) for the Australian economy, adopted an AR(2) process for the output gap. Our estimates rejected it in favor of an AR(1) process for Australia.

to boost the output gap temporarily. As there appears to be a lag of one year for the change to affect the output gap, we include the fourth lag of the quarterly change of the (logged) terms of trade ($\Delta LTOT_{t-4}$). Given Australia is a small open economy, influences of foreign activity on the home country feed through the contemporaneous quarterly change of the G7 output gap ($\Delta \tilde{y}_t^{G7}$). In the medium run, the output gap converges to zero as do the real interest rate gap, the G7 output gap, and changes in the terms of trade.

The aggregate supply side of the economy is represented by an expectations-augmented Phillips curve (4):

$$\pi_t - \pi_{t-1} = b_1 \tilde{u}_t + b_2 \tilde{\pi}_{t-1}^{imp} + b_3 \tilde{\pi}_t^e + \varepsilon_t^\pi \tag{4}$$

where inflation expectations are assumed to be driven by backward- and forward-looking processes. Economic agents rely on past inflation information as well as future information to condition their inflation forecasts, hence the inclusion of the lagged inflation rate and consumer survey expected inflation advocated by Roberts (1997). The influence of excess demand on inflation is captured by the unemployment gap ($\tilde{u}_t$), which reflects the nominal inertia of wage responses to economic activity, which are marked up into prices. The pass-through effect on to domestic inflation of import prices represents a supply factor, which enters the equation with a lag. More specifically, consumer survey inflation expectations and import price inflation are constructed to be zero in the medium run. The variable $\tilde{\pi}_t^e$ is the excess of consumer inflation expectations over lagged year-ended inflation, i.e. $\tilde{\pi}_t^e = \pi_t^e - \pi_{t-1}$, while $\tilde{\pi}_t^{imp}$ is the excess of import price inflation over lagged year-ended inflation, $\tilde{\pi}_t^{imp} = \pi_t^{imp} - \pi_{t-1}$ (see Gruen et al., 2005). In the absence of supply shocks, (4) yields a vertical Phillips curve in the medium run, with output anchored at its potential level, and actual equal to expected inflation.[6]

The connection between the unemployment gap and the output gap is represented by an Okun equation (5):

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[6] Gordon (1997) observed that the estimated sum of the lagged inflation terms must be constrained to unity so that meaningful natural output or unemployment can be derived.

$$\tilde{u}_t = c_1 \tilde{u}_{t-1} + c_2 \tilde{u}_{t-2} + c_3 \tilde{y}_t + \varepsilon_t^{\tilde{u}} \tag{5}$$

where some degree of persistence in the dynamics of the unemployment gap is captured by an AR(2) process.

Equations (6) through (11) describe the laws of motion of the unobservable trends in the model. Potential output is modeled by (6) as a local linear trend, where the drift term μ_{t-1} representing the trend growth rate is a random walk process (7):[7]

$$y_t^* = y_{t-1}^* + \mu_{t-1} + \varepsilon_t^{y*} \tag{6}$$

$$\mu_t = \mu_{t-1} + \varepsilon_t^{\mu} \tag{7}$$

Gruen et al. (2005) found large shifts in the trend growth rate for the Australian economy since 1960. To incorporate this feature of the Australian economy, the local linear trend specification implies that potential output grows at the time-varying normal growth rate when all shocks dissipate in the medium run, i.e. $\Delta y_t^* = \mu_{t-1}$.[8]

The relationship governing movements of the natural real interest rate is derived from intertemporal household maximization. Consider an infinite horizon representative agent model, where the intertemporal utility is $U_t = \sum_{s=t}^{\infty} \beta^{s-t} C_s^{1-1/\sigma}$, with σ denoting the intertemporal elasticity of substitution and β the rate of time preference. The first-order condition of the optimal consumption is

$$\frac{\mu_{c,t}}{\sigma} = \ln \beta + \ln(1 + r_t) \tag{8}$$

where $\mu_{c,t}$ is the growth rate of consumption. We do not insert the implied restrictions from this intertemporal optimizing condition into the short-run product market equilibrium condition (3) on the grounds that many households are unable to optimize in the short run. However in the medium run, most will find a way to approach their optimal consumption

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[7] Since the drift term is assumed to be I(1), this implies that potential output and log real GDP are I(2). This hypothesis is typically rejected by an ADF test. However, Stock and Watson (1998) pointed out that the test statistic tends to have high probability of type I error in falsely rejecting the true null when the variance of the trend growth rate innovation is small.

[8] Alternatively the trend growth equation (7) can be modeled as an autoregressive process that makes y^* and y difference-stationary. However, the sum of the autoregressive parameters obtained during preliminary estimations suggests that it is almost identical to unity and hence highly persistent.

trajectory. In the medium-run equilibrium of the open economy, the current account to GDP ratio will be constant, as will be the consumption to GDP ratio, i.e. $\mu_{c,t} = \mu_t$. Therefore, at low interest rates, the medium-run relation between the real interest rate and output growth is approximated by $r_t = \mu_t / \sigma - \ln \beta$. We therefore model the natural rate of interest, r_t^*, according to the medium-run optimal condition. Equation (9) shows that d represents the inverse of the elasticity of intertemporal substitution, and the log of the rate of time preference, $\ln\beta_t$, is a random walk process (10).[9]

$$r_t^* = d\mu_t - \ln \beta_t \qquad (9)$$

$$\ln \beta_t = \ln \beta_{t-1} + \varepsilon_t^\beta \qquad (10)$$

Lastly, the underlying factors that determine the natural rate of unemployment in the labor market are assumed to be reasonably well modeled by stochastic trends. Therefore the natural rate of unemployment follows a random walk, which is the standard specification in other studies to capture time variations in this unobservable macroeconomic series.

$$u_t^* = u_{t-1}^* + \varepsilon_t^{u*} \qquad (11)$$

To complete the description of the multivariate UC model, we assume that all innovations $\varepsilon = \left(\varepsilon_t^{\tilde{y}}, \varepsilon_t^{\pi}, \varepsilon_t^{\tilde{u}}, \varepsilon_t^{y*}, \varepsilon_t^{\mu}, \varepsilon_t^{\beta}, \varepsilon_t^{u*} \right)'$ are i.i.d. normally distributed with zero mean and finite variances. In addition, they are serially and contemporaneously uncorrelated with each other.

4. Data and empirical results

4.1 Data

The quarterly data span starts from 1984:1 to 2006:4. All data unless otherwise specified were obtained from the Australian Bureau of Statistics. y_t and u_t are the Australian real GDP and unemployment rate. Domestic inflation is calculated as the year-ended change in the log of

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[9] Laubach and Williams (2003) considered an AR(2) process, and Garnier and Wilhelmsen (2005) an AR(1). However, these alternative specifications did not generate economically sensible results for the Australian economy and would additionally violate the identification condition discussed in section 4.2.

the headline CPI. The same procedure is applied to compute the import inflation rate based on the log of the import chain price index. The real interest rate is the nominal cash rate less the inflation rate, i.e. $r_t = i_t - \pi_{t+1}$. Quarterly changes in the log of the terms of trade index are used since they offer higher explanatory power in the preliminary OLS estimation. Inflation expectations are consumers' inflation expectations measured by the Melbourne Institute as the median expected inflation rate for the year ahead. Estimates of the G7 output gap are extracted from the OECD database.

4.2 Estimation issues

Before proceeding with estimation, the multivariate UC model is cast in the state-space form (see the appendix). Parameters are estimated by maximum likelihood as described in Harvey (1989). Their initial values are drawn from OLS regressions (see Hamilton, 1994). Natural rates, or state variables, are simultaneously extracted using the Kalman filter. The Kalman filter is a recursive algorithm that sequentially updates a linear projection of a dynamic system. In each period the Kalman filter provides the (one-sided) optimal predictions of the natural rates for that period conditional on information available up to and including the current period. Once the filtered natural rates are obtained, it is possible to 'smooth over' the natural estimates conditioned on information from the full sample; therefore the smoothed natural rates can be thought of as two-sided estimates. There are two important estimation issues related to the Kalman filter that need to be resolved: namely the choice of the initial values of the state vector and covariance matrix and the estimation of the innovation variances.

To set the initial values for the state vector, the gap measures are assumed to be stationary, and so a value of zero is assigned to them. For the natural rate measures, the initial value is set to the value of the first observation of the associated variable. The dynamics of the multivariate UC model are non-stationary because the trend equations are specified to be random walks. Therefore we follow the usual practice of assigning diffuse priors to the

diagonal elements of the initial state covariance matrix.[10]

It is common in the literature to choose a value for the signal-to-noise ratio and impose it in the maximum likelihood estimation. One example in our model is the ratio of trend growth innovation to potential output innovation, $\lambda = \sigma_\mu / \sigma_{y*}$. Because these two unobserved variables are non-stationary, their cumulated variance goes to infinity, and so the ratio of their maximum likelihood estimates (MLE) has a point mass at zero even though their true values are greater than zero. Stock (1994) discussed this so-called 'pile-up' problem which prevents the efficient estimation of the innovation variance of the non-stationary state variables. To circumvent the pile-up problem, Laubach and Williams (2003) applied the median-unbiased estimation procedure developed by Stock and Watson (1998). The first step is to obtain the median-unbiased estimates of the signal-to-noise ratio. In the second stage, the ratio is imposed in the system estimation. Laubach and Williams (2003) and Garnier and Wilhelmsen (2005) followed this approach to estimate potential output and the natural real interest rate for the U.S. and the Euro zone respectively.

In recognition of the pile-up problem, Messonier and Renne (2004) argued that it becomes difficult to pin down a sensible path of the natural real interest rate, because r^* is an unobserved process that is linked to two other unobserved processes, μ and $\ln\beta$. Instead, they followed the approach in King et al. (1995), Staiger et al. (1996), and Laubach (2001) in fixing the signal-to-noise ratio at particular values and testing them statistically in reference to a baseline model.

We offer another perspective to the need for fixing the values of the unconditional variances of innovations through the point of view of parameter identification. By first-differencing the potential output equation (6) and the trend growth equation (7), we get:

$$\Delta y_t^* = \mu_{t-1} + \varepsilon_t^{y*} \tag{12}$$

$$\Delta \mu_t = \varepsilon_t^\mu \tag{13}$$

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[10] We found it necessary to begin the recursion with a diffuse prior of zero to tie the estimated trends to a path that would run through the data. The result is that the filtered and smoothed estimates are very close to the first observation of the variable.

where according to (12) and (13) the two structural parameters to be identified are σ^2_{y*} and σ^2_{μ}.

Lag (13) by one period to obtain:

$$\mu_{t-1} = \frac{\varepsilon^{\mu}_{t-1}}{\Delta} \tag{14}$$

Substitute (14) into (13) yields:

$$\Delta y^*_t = \frac{\varepsilon^{\mu}_{t-1}}{\Delta} + \varepsilon^{y*}_t$$

$$\text{or} \tag{15}$$

$$\Delta^2 y^*_t = \varepsilon^{\mu}_{t-1} + \varepsilon^{y*}_t - \varepsilon^{y*}_{t-1}$$

The autocovariance functions of the reduced-form equation $\Delta^2 y^*_t$ are

$$\gamma(0) = \sigma^2_{\mu} + 2\sigma^2_{y*}$$
$$\gamma(1) = -\sigma^2_{y*} \tag{16}$$
$$\gamma(\tau) = 0 \quad \text{for } \tau \geq 2$$

where $\gamma(\tau)$ is the τ-th order autocovariance function. Given (16), we have two reduced-form parameters to map to two structural parameters. Hence the order condition is satisfied to identify the structural parameters σ^2_{y*} and σ^2_{μ}.

Now consider the r^* and $\ln\beta$ equations (9) and (10). Substitute out μ_t and $\ln\beta_t$ with $\mu_t = \varepsilon^{\mu}_t/\Delta$ and $\ln\beta_t = \varepsilon^{\beta}_t/\Delta$ to yield the following reduced form equation of Δr^*_t:

$$\Delta r^*_t = d\varepsilon^{\mu}_t - \varepsilon^{\beta}_t \tag{17}$$

We need to identify from (17) the structural parameters d and σ^2_{β} with σ^2_{μ} already identified previously in (16). The autocovariance functions of Δr^*_t are

$$\gamma(0) = d^2\sigma^2_{\mu} + \sigma^2_{\beta}$$
$$\gamma(\tau) = 0 \quad \text{for } \tau \geq 1 \tag{18}$$

Therefore in this case we have an under-identification problem as there is only one reduced-form parameter available to link to the two structural parameters d and σ^2_{β}. In relation to previous studies that fix the unconditional variances of state variable innovations through the signal-to-noise ratio, we find the problem arises here with σ^2_{β} and not σ^2_{μ}. Since the parameter d approximates the inverse of the intertemporal elasticity of substitution,

we calibrate σ_β with a range of values and discard those that do not generate significant estimates of d.[11]

4.3 Results

The calibrated values of $\sigma_\beta = 0.6$ to 1.6 yield significant estimates of d ranging from 5.64 to 6.36. This approximates to a range for the intertemporal elasticity of substitution (σ) between 0.16 and 0.18. We deem these to be reasonable estimates as Barsky et al. (1997) using micro-data came up with an estimate of 0.18.

Table 1 displays the parameter estimates of the multivariate UC model. Kalman smoothed estimates of potential output and its normal growth rate, the NAIRU, the natural real interest rate and their related gap measures, plus the implied non-constant rate of time preference are shown in Figures 1 to 8.

All of the estimated coefficients have the expected sign. The sum of the autoregressive parameter estimates of the IS equation (3) and the Okun equation (5) are each less than one, which indicates stationary dynamics of the output and unemployment gaps.

[---- insert Table 1 here ----]

In all of the figures, the UC natural rates are the smoothed estimates associated with σ_β = 1.1 as it yields the highest significance for d. The multivariate unobserved components (MUC) measure (in blue) is compared to a univariate measure (in red) derived from the band-pass filter (BP). We use BP for comparison because it is a reasonably efficient univariate filter, but the comparisons would be largely unaffected if we had chosen another candidate (such as the Hodrick-Prescott filter). The shaded space above and below the multivariate measures represent 95% confidence intervals. These were obtained through 5000 Monte Carlo conditional simulations that compute the second moment of the Kalman smooth states using

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[11] These are $\sigma_\beta = (0.001, 0.6, 0.8, 1.0, 1.1, 1.2, 1.4, 1.6, 3.0)$.

Gibbs sampling. The resulting upper and lower bands represent the effects of both parameter and filter uncertainty (Hamilton, 1986).

In Figures 1 and 2, the MUC pattern of potential output indicates a brief period of expansion at the end of the 1980s. This is followed by a period of excess capacity covering much of the 1990s. In comparison to the BP potential output measure, the MUC measures suggest that the Australian economy headed into contraction in 1990Q3, earlier by two quarters. In addition, the MUC contraction is steeper and more persistent, attaining its trough in 1992Q2 at -6.6% as opposed to the BP at -2.5%. Towards the end of the sample period, however, the MUC output gap disagrees with the BP measure, suggesting that the economy was in a period of growing excess demand to 1.5% (though the 95% confidence interval just includes 0). This shows the merits of conditioning the path of potential output by incorporating information from the aggregate supply side through the Phillips curve and Okun's law, and from the aggregate demand side through the dynamic IS curve with intertemporal optimization.

[---- insert Figure 1 here ----]

[---- insert Figure 2 here ----]

We show in Figure 3 that the MUC measure of the NAIRU fell throughout the sample period in general, except for the temporary and minor pickup around the 1990-91 recession. Interestingly, the MUC NAIRU declined fastest in the 1980s, which suggests that was the substantive decade of labor market reform. Unlike the BP filter which essentially plots the trend line through the unemployment data, the multivariate NAIRU was much lower for most of the time. At the end of 1996, the NAIRU was down to 6.7% (with a 95% confidence interval of {4.7%,8.6%}). Even though the unemployment rate has been on a downward trend since the early 1990s, the result suggests that the accompanying slower decline in the NAIRU has buffered the Australian economy from inflationary pressure. At the end of 2006, it had

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come down to 5.4% with a confidence interval of {2.7%,8.1%}. Mirroring the estimates of the output gap, the MUC unemployment gap values in Figure 4 show that the slack conditions in the labor market persisted for much of the 1990s, despite the BP measure suggesting tightness after 1994. At the end of 2006, unlike the BP filter, the MUC measure indicates that the labor market had become increasingly tight with a gap measure of -0.82% (though the 95% confidence interval did just include 0).

[---- insert Figure 3 here ----]

[---- insert Figure 4 here ----]

The BP and MUC filter provide contrasting perspectives on the real interest rate, and by implication on the monetary policy stance over the sample period. As seen in Figures 5 and 6, the univariate BP estimates suggest that the natural real interest rate was much higher than the MUC estimates until 1996, but the roles reverse in the 2000s. The BP real interest rate gap measure indicates that monetary policy became expansionary from 1987 after the stock market crash and for about three years after the recession in 1991, but was only modestly restrictive (+4.9%) between the two expansionary phases. Given the depth of the early 1990s recession, this is an unsatisfactory result. On the other hand, the MUC measure suggests that monetary policy was highly contractionary (peaking at a +12.7% real interest rate gap, beginning 1990), only to be reversed into expansion in 2000. During the severe monetary policy contraction in 1989-90, the actual real cash rate went up dramatically, and the BP natural real rate measure followed it up to a degree in its economic blindness. By contrast, in 1989-90, the MUC real rate fell, which is what economic insight would suggest. Since actual and normal output and consumption growth fell in that recession, the medium-run real interest rate had to follow suit to a degree to maintain intertemporal balance. Since 2000, the MUC real interest rate gap suggests monetary policy has stayed relatively stimulative with the natural real interest rate converging on 4.1% at the end of 2006. However the 95% confidence

interval is wide indicating the difficulty in obtaining precise estimates of unobserved real interest rates. This result underscores the caution exhibited by policymakers when making monetary policy decisions.

[---- insert Figure 5 here ----]

[---- insert Figure 6 here ----]

The estimates of the underlying normal quarterly growth rate of the economy are shown in Figure 7. The BP estimates have more exaggerated movements. The MUC estimates show a general fall in normal growth rates over the last 22 years from 4.3% to 2.4%. The fall steepened with the onset of the 1990-1 recession, but turned at the bottom of that recession, rising until the end of 1996. The normal rate of growth has actually declined significantly since 1996, despite output growth being consistently stable and positive. At the end of 2006, the normal growth rate was 2.4%, surprisingly lower than what it had been in the 1991 trough (3.1%). However the 95% confidence interval was wide, and again this is a recommendation for added caution in policy design.

The general fall in the normal growth rate (even when accounting for the confidence intervals) is a matter of concern. It may be construed as a serious indictment of the economic management by government (both Labor until 1996, and then Coalition), which has failed to arrest the decline in underlying productivity growth. The efficiency gains of recent policy initiatives would seem to be more about pushing the economy to work above potential, rather than trying to reverse the decline in the normal growth rate. These results suggest that Australia needs better long term management to improve physical infrastructure, R&D incentives, productive investments well beyond housing, information and communications technologies, and the productivity of education.

[---- insert Figure 7 here ----]

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Finally, Figure 8 presents our results for the non-constant time preference factor. This was modeled as a random walk, with a given variance in equation (10). The 95% confidence interval includes values below 1 in all periods apart from around 1990. Economic models with an endogenous discount factor, e.g. described in Schmitt-Grohe and Uribe (2003), states that agents become increasingly impatient when consumption growth rises. This last result suggests that future utility is valued more highly when the economy is not performing so well during the contractionary 1990s that included the recession at the beginning, which should lead to a rise in the observed saving rate. In the 2000s, the point estimate for the time preference factor converged on a value just below unity.

[---- insert Figure 8 here ----]

5. Concluding comments

Natural rates and normal growth rates are medium-run benchmarks that permit a judgement about whether the actual rates are too high, too low or just right. We have jointly estimated the time paths of these unobservable benchmarks using maximum likelihood methods with the Kalman filter for Australian data from 1984 to 2006. We constructed a standard macroeconomic model for inflation, output and unemployment, and our parameter estimates were all significant (but one) with the expected signs. From the inferred natural rate of unemployment, the natural level of output (or potential output) and its normal growth rates, and the natural real rate of interest, we have been able to assess the state of the actual economy and comment on the stance of monetary policy over the two decades of the sample.

We find that our multivariate unobserved components model generates results that have far more economic significance than a univariate band-pass filter. We provide Monte Carlo simulated 95% confidence intervals for our estimates of the time paths of the unobservable natural rate variables. These intervals are typically wide, and this leads us to conclude that policy makers wisely practice caution when designing their monetary and fiscal policy

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responses.

At the end of 2006, we conclude that output was just above potential (+1.5%), that its normal quarterly growth rate was actually quite low at about 2.4% on an annualized basis, that unemployment was about 0.8 percentage point below its natural rate of 5.4%, and that the real cash rate was actually 0.2 percentage points below its natural rate of 4.1%. This suggests that monetary policy was possibly still expansionary, but it is not clear that this could have any beneficial effect with output above potential, and with monetary policy expected to be neutral in regard to the normal rate of growth. Correcting the downward trend in the normal growth rate of GDP is a major challenge and will require more than monetary and fiscal policy responses by government. The decline in underlying productivity growth can be arrested only by policies designed to address long-run efficiencies. Estimates for the time preference factor suggest that the sluggish economic performance in the 1990s induced people to increase savings in view of higher utility on future consumption. As the economy improved and monetary policy became increasingly expansionary from 2000, higher valuation was apparently placed on current consumption as people became less patient.

Appendix

The state space representation is consisted of a measurement equation:

$$\mathbf{w}_t = \mathbf{F}\boldsymbol{\xi}_t + \mathbf{v}_t \tag{A.1}$$

and a transition equation:

$$\boldsymbol{\xi}_t = \mathbf{G}\boldsymbol{\xi}_{t-1} + \mathbf{H}\mathbf{x}_t + \boldsymbol{\upsilon}_t \tag{A.2}$$

where $\mathbf{w}_t$ is the vector of observable variables, $\boldsymbol{\xi}_t$ is the vector of state (or unobservable variables), and $\mathbf{x}_t$ is the vector of exogenous variables. $\mathbf{v}_t$ and $\boldsymbol{\upsilon}_t$ are white noise innovation vectors.

The measurement equation in matrix form:

$$
\begin{bmatrix} y_t \\ u_t \\ \pi_t \end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\tilde{y}_t \\ \pi_t \\ \pi_{t-1} \\ \tilde{u}_t \\ \tilde{u}_{t-1} \\ y_t^* \\ \mu_t \\ r_t^* \\ r_{t-1}^* \\ r_{t-2}^* \\ r_{t-3}^* \\ r_{t-4}^* \\ r_{t-5}^* \\ r_{t-6}^* \\ r_{t-7}^* \\ u_t^* \\ \ln \beta_t
\end{bmatrix}
$$

The transition equations in matrix form:

$$
\begin{bmatrix} \tilde{y}_t \\ \pi_t \\ \tilde{u}_t \\ \tilde{u}_{t-1} \\ y_t^* \\ \mu_t \\ r_t^* \\ r_{t-1}^* \\ r_{t-2}^* \\ r_{t-3}^* \\ r_{t-4}^* \\ r_{t-5}^* \\ r_{t-6}^* \\ r_{t-7}^* \\ u_t^* \\ \ln \beta_t \end{bmatrix}
=
\begin{bmatrix}
a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -a_2 & 0 & 0 \\
b_1 c_3 a_1 & 1 & b_1 c_1 & b_1 c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_1 c_3 a_2 & 0 & 0 \\
c_3 a_1 & 0 & c_1 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_3 a_2 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} \tilde{y}_{t-1} \\ \pi_{t-1} \\ \tilde{u}_{t-1} \\ \tilde{u}_{t-2} \\ y_{t-1}^* \\ \mu_{t-1} \\ r_{t-1}^* \\ r_{t-2}^* \\ r_{t-3}^* \\ r_{t-4}^* \\ r_{t-5}^* \\ r_{t-6}^* \\ r_{t-7}^* \\ r_{t-8}^* \\ u_{t-1}^* \\ \ln \beta_{t-1} \end{bmatrix}
$$

$$
+
\begin{bmatrix}
a_2 r_{t-8} + a_3 \Delta LTOT_{t-4} + a_4 \Delta \tilde{y}_t^{G7} \\
b_1 c_3 \left(a_2 r_{t-8} + a_3 \Delta LTOT_{t-4} + a_4 \Delta \tilde{y}_t^{G7} \right) + b_2 \tilde{\pi}_{t-1}^{imp} + b_3 \tilde{\pi}_t^e \\
c_3 \left(a_2 r_{t-8} + a_3 \Delta LTOT_{t-4} + a_4 \Delta \tilde{y}_t^{G7} \right) \\
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
+
\begin{bmatrix}
\varepsilon_t^{\tilde{y}} \\
b_1 c_3 \varepsilon_t^{\tilde{y}} + \varepsilon_t^{\pi} \\
c_3 \varepsilon_t^{\tilde{y}} + \varepsilon_t^{\tilde{u}} \\
0 \\
\varepsilon_t^{y^*} \\
\varepsilon_t^{\mu} \\
d \varepsilon_t^{\mu} + \varepsilon_t^{\beta} \\
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \varepsilon_t^{u^*} \\ \varepsilon_t^{\beta}
\end{bmatrix}
$$

The covariance matrix of the residuals of the transition equations is as follows:

$$
Q = \begin{bmatrix}
\sigma_{\bar{y}}^2 \\
b_1 c_3 \sigma_{\bar{y}}^2 & b_1 c_3 \sigma_{\bar{y}}^2 + \sigma_{\pi}^2 \\
c_3 \sigma_{\bar{y}}^2 & b_1 c_3^2 \sigma_{\bar{y}}^2 & c_3^2 \sigma_{\bar{y}}^2 + \sigma_u^2 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_{y*}^2 \\
0 & 0 & 0 & 0 & 0 & \sigma_{\mu}^2 \\
0 & 0 & 0 & 0 & 0 & d\sigma_{\mu}^2 & d^2\sigma_{\mu}^2 + \sigma_{\beta}^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{u*}^2 \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\beta}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\beta}^2
\end{bmatrix}
$$

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Table 1: Parameter Estimates of the Multivariate UC Model

Parameters	σ_β=0.6	σ_β=1.1	σ_β=1.6
a_1	0.80	0.66	0.53
	0.09***	0.13***	0.19***
a_2	-0.11	-0.16	-0.18
	0.05**	0.06***	0.07**
a_3	0.09	0.09	0.08
	0.03***	0.03**	0.04**
a_4	0.86	0.78	0.67
	0.20***	0.21***	0.22***
b_1	-0.21	-0.20	-0.18
	0.07***	0.06***	0.07***
b_2	0.03	0.03	0.03
	0.01***	0.01***	0.01***
b_3	0.18	0.17	0.16
	0.05***	0.05***	0.05***
c_1	1.41	1.42	1.43
	0.11***	0.11***	0.11***
c_2	-0.60	-0.61	-0.61
	0.09***	0.09***	0.09***
c_3	-0.11	-0.11	-0.10
	0.02***	0.02***	0.02***
d	5.64	6.20	6.36
	0.69**	0.57***	0.81**
Standard Errors of Shocks			
$\sigma_{\bar{y}}$	0.53	0.46	0.38
	0.05***	0.06***	0.09***
σ_{π}	0.72	0.73	0.73
	0.06***	0.06***	0.06***
$\sigma_{\bar{u}}$	0.07	0.07	0.07
	0.02***	0.02***	0.02***
σ_{y*}	0.00	0.00	0.00
	0.15	0.15	0.12
σ_{μ}	0.05	0.05	0.05
	0.01***	0.01***	0.01***
σ_{u*}	0.15	0.15	0.14
	0.02***	0.02***	0.02***
Loglikelihood	-150	-148	-147

Note: Standard errors are given below each estimate .

* designates significance at 10%, ** at 5%, and *** at 1%.

Figure 1
y: Real GDP and y^*: Potential Output

ACTUAL — POTENTIAL — Band Pass

Figure 2
y-y^*: Potential GDP Gap
(& 95% Confidence Interval)

GDP GAP ----- UPPER ----- LOWER — Band Pass

Figure 3
u*: Unemployment - NAIRU
(& 95% Confidence Interval)

NAIRU ····· UPPER ····· LOWER —— Actual —✻— Band Pass

Figure 4
u -u*: Smoothed Unemployment Gap
(& 95% Confidence Interval)

—— UNEMPLOYMENT GAP ····· UPPER ····· LOWER —✻— Band Pass

Figure 5
r*: Natural Real Interest Rate
(& 95% Confidence Interval)

Figure 6
r-r*: Real Interest Rate Gap
(& 95% Confidence Interval)

Figure 7
μ: **Normal GDP Growth Rate**
(& 95% Confidence Interval)

NORMAL GROWTH RATE ---- UPPER ----- LOWER --✱-- Band Pass

Figure 8
β: **Time Preference Factor**
(& 95% Confidence Interval)

----- UPPER ----- LOWER ━━ BETA